A Stochastic Dynamic Programming Model of Ancillary Storage Using Electric Vehicles to Offset Volatility from Wind Generation

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Submitted in partial fulfillment of the requirements for the degree of Bachelor of Science in Engineering Department of Operations Research and Financial Engineering Princeton University

June 2011
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Ben Sheng
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Abstract

As wind generation expands toward the goal of 20% electricity from wind by 2030, the problem of wind volatility will become more serious. Plug-in hybrid electric vehicles offer a significant source of stored energy that utilities are interested in using to offset the variability of energy from wind turbines. The thesis formulates the problem of when to charge and discharge batteries for electric vehicles in a region, modeled as a single energy aggregator. The model is solved using stochastic dynamic programming to find an optimal policy for charging and discharging, and the model is then used to develop an understanding of the number of cars needed to balance a set of wind turbines.
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Chapter 1. Introduction

The United States consumed 95 quadrillion BTU of energy in 2009, of which 79 quadrillion (or 83%) came from traditional fossil fuels (U.S. Energy Information Administration, 2010). However, coal and petroleum are in limited supply, are subject to frequent price shocks, release large amounts of greenhouse gases, and threaten America’s national security as the country is dependent on imports of oil. Renewable energy currently supplies only 8% of total U.S. energy consumption, but could be a larger part of the energy solution that mitigates global warming and reduces
America's reliance on foreign fossil fuels (U.S. Energy Information Administration, 2010).

Wind is one of the fastest growing sources of renewable energy, and in 2008, the U.S. Department of Energy announced a goal for wind energy to provide 20% of the US electricity supply by 2030 (U.S. Department of Energy, 2008). In addition, wind is also gaining the attention of the business community. In October 2010, Google and Good Energies, a New York investment firm specializing in renewable energy, committed to investing in a proposed $5 billion underwater transmission line for future offshore wind farms along the mid-Atlantic coast. This 350-mile line will greatly simplify the construction of offshore wind farms and reduce their operating cost (Wald, 2010).

![Figure 1-1: US Annual Average Wind Power](image)

Wind power is widely available in the US as shown in the blue areas of Figure 1-1. However, it depends on a resource (wind speed) that is highly volatile and varies
by time of day, season, and weather. At high wind speeds, more energy is produced than demanded, and the excess energy is wasted. At low wind speed, little or no energy is produced. When wind is a small proportion of the nation’s energy portfolio, production at coal-fired or natural gas-fired power plants can shift slightly to compensate for wind volatility. However, if wind is to provide 20% of the nation’s electricity, this type of balancing has little use. What is needed is a form of electricity storage in order to respond to unpredictable output or demand (Schoppe, 2010).

Figure 1-2: Chevrolet Volt, an Example PHEV

By “storage,” more formally known as ancillary storage, this thesis means holding energy produced during times of high supply/low demand and releasing it during periods of low supply/high demand. According to Professor Powell, one promising form of storage is using batteries from plug-in hybrid electric vehicles (PHEVs) shown in Figure 1-2 and electric vehicles (EVs) shown in Figure 1-3. (In
the rest of the thesis, we will mainly refer to PHEVs.) The electric utility would sign a contract with PHEV drivers, and pay either for the right to draw electricity from the car battery or for the amount of electricity used. Batteries are too expensive for wind farm operators and utilities to purchase, but in this case, the driver has already paid the high fixed cost of the battery. Through this agreement, PHEV owners would be able to recoup part of the cost of owning a hybrid, and if enough drivers participate, the utilities could effectively use the car batteries as storage. This would improve the viability of both wind energy and PHEVs resulting in additional benefits of energy security and greenhouse gas reduction.

![Nissan Leaf, an Example EV](image)

Figure 1-3: Nissan Leaf, an Example EV
1.1 Thesis Overview

For wind to provide 20% of the nation’s energy by 2030, intermittency of wind output must be balanced. Energy storage can solve this problem, and this thesis studies the feasibility of using PHEVs as the storage to offset wind volatility.

In Chapter 2, a summary of the data used in the simulation of PHEVs as grid storage is given, and this includes wind speed, electricity spot prices, demand, driving patterns, and plug-in patterns. Chapter 3 provides the model of the power system where the electric utility makes decisions to minimize the cost of supplying electricity to demand. A wind farm serves as the primary energy source, and PHEVs are the main storage mechanism. Chapter 4 discusses how the model is implemented, and includes simplifying assumptions that make the model computable and values for the model parameters. A stochastic dynamic programming algorithm is used to find the optimal charging or discharging decision at each state. Chapter 5 presents the results from the simulations, and shows how varying parameters changes the benefit of being able to discharge vehicle batteries. In the conclusion Chapter 6, the implications of our research are discussed, and areas of possible future research are pointed out.
Chapter 2. Data

In order to create simulations of PHEV batteries balancing wind generation, real data is used for wind speed, spot market price, and demand, and PHEV driver behavior is also modeled.

2.1 Wind Data

In this paper, we collect wind speed data and convert that to power. Wind from offshore New Jersey is the focus.

2.1.1 Available Power from Wind

The power available in the wind is a function of the cube of wind speed as shown below in Equation 2-1. The power output $P$ in watts ($\text{kg m}^2/\text{s}^3$) from turbines is given by:
\[ P = \frac{1}{2} B \rho A v^3 \]  

(2-1)

where \( \rho \) is the density of air (around 1.225 kg/m\(^3\) at sea level), \( B \) is the wind turbine’s power coefficient describing the fraction of wind captured that can be converted into mechanical energy, \( A \) is the area swept by rotor blades in m\(^2\), and \( v \) is the wind speed in m/s. The Betz limit gives a theoretical maximum of \( B \) at \( \frac{16}{27} = 0.593 \), or about 59.3\%. This means that the wind turbine can use at most 59.3\% of the available wind power. Because of the cubic relationship between power generation and wind speed, slight changes in wind speed can result in vastly different outputs. For instance, all else being equal, an increase in wind speed from 6 mph to 7 mph would lead to about 59\% more power output \( \left( \frac{7^3}{6^3} = 1.59 \right) \) (Wei, 2010). One could increase power generated by making the turbines more aerodynamic (increasing \( B \)), by lengthening the rotor blades (increasing \( A \)), or by choosing a location with higher average wind speeds (increasing \( v \)).

Most wind turbines begin generating electricity at wind speeds around 2-4 m/s (8 mph) and shut down to prevent storm damage at 25 m/s (50 mph) or above. Most turbines reach maximum power around 15 m/s (30 mph) (BWEA, 2005), and this output is the value commonly quoted as the “installed capacity” of a wind turbine ("The Capacity Factor of Wind Power").
Offshore wind turbines are typically between 2 and 4 MW ("Offshore Wind Energy"), and we assume the higher end of 4 MW for this thesis. Using the equation above, we have \( P = C v^3 \), where \( C = 1185 \text{ W/(m/s)}^3 \).

2.1.2 Analyzing Wind Data

This study utilizes a database from NLDAS with hourly wind data from October 1996 to December 2005. The wind speed was measured 10m from the ground, broken into North-South and East-West components, and given in units of m/s. The data set is organized in a grid structure and includes 179 locations across the continental US with latitude ranging from 25.1875N to 48.9375N and the longitude ranging from 67.5625W to 124.9375W.

This paper uses wind data from an offshore South New Jersey location at 39.6875N 74.0625W as shown in Figure 2-1 below. We want to study wind energy in NJ, and the most significant region of wind production here is offshore.

![Figure 2-1: The Offshore South New Jersey Location](image)
In Figure 2-2, we graph the hourly wind speeds of 2005, and use Equation 2-1 to create a wind generation plot. Wind is highly volatile especially in power generation, and there also appears to be seasonality in wind.

Figure 2-2: 2005 Hourly Wind Data
Indeed, considering wind speed data for winter 2005 (January 1\textsuperscript{st} through March 31\textsuperscript{st}) in Figure 2-3 and data for summer 2005 (June 1\textsuperscript{st} through August 31\textsuperscript{st}) in Figure 2-4, winter winds tend to be at higher speeds than summer winds. The median winter wind speed is 5.88 m/s compared to 4.78 m/s for summer, and is 85% more powerful. Winter winds also tend to be more volatile with a standard deviation of 2.90 m/s compared to 1.85 m/s for summer. However, the simulation does not consider seasonality in wind due to time constraints.
There seems to be diurnal variation in wind speed in Figure 2-5 based on wind data for the single month of December 2005. However, based on Figure 2-6 which shows the hourly wind speeds for each day, wind speeds appear to be better correlated with other hours of the same day and poorly correlated with the same hour of a different day. Thus, rather than try to forecast wind in advance, we find the probability of wind speed being at a certain value given wind speed in the previous hour, and this model is used to generate sample hourly wind profiles in the simulation.
In Figure 2-8, we graph hourly wind speed and power generation from the South Jersey location and from an offshore Massachusetts location at 41.5625N 70.3125W shown in Figure 2-7. Looking at wind output data from this sample day of December 28, 2005, it appears that using wind farms from multiple locations reduces the intermittency of wind. While South New Jersey and Massachusetts each have 5 hours of zero wind generation, the combined power output falls to zero for only 2 hours. Due to time constraints, we only use the South New Jersey wind data, which suggests that our simulated wind output is more volatile than reality.
As a point of comparison, we also analyze data from a North New Jersey location at 40.6875N 74.0625W as shown in Figure 2-9 below.
To measure the average wind generation at the 3 locations, we use capacity factor, which is the ratio of electrical energy produced in a given period of time to the electrical energy that could have been produced at continuous maximum power generation (i.e., between 15 m/s and 25 m/s wind speed) ("The Capacity Factor of Wind Power"). In Table 2-1, we contrast the median wind speed and capacity of the 3 locations from October 1996 to December 2005. Offshore Massachusetts is the best location while North New Jersey is the worst perhaps because it is too far inland to receive the stronger shore winds. The offshore South New Jersey location is more reliable than North New Jersey, but less reliable than offshore Massachusetts.

<table>
<thead>
<tr>
<th>Location</th>
<th>Median Wind Speed</th>
<th>Capacity Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>North New Jersey</td>
<td>3.72 m/s</td>
<td>4.49%</td>
</tr>
<tr>
<td>Offshore South New Jersey</td>
<td>5.15 m/s</td>
<td>8.76%</td>
</tr>
<tr>
<td>Offshore Massachusetts</td>
<td>5.70 m/s</td>
<td>12.05%</td>
</tr>
</tbody>
</table>
2.2 Regulated Market Data

The spot market, more formally known as the regulated market (RM), is where the utility can purchase electricity for immediate delivery. For RM price data, we use hourly real time locational marginal price (price of electricity charged by generators to retailers) data from PJM West.

Graphing the spot market price by hour in Figure 2-10, there appears to be high hourly variation in price, and some days have more price volatility than others. However, in this thesis, we do not attempt to simulate RM price by hour as we do for wind because of programming constraints.

![Figure 2-10: Hourly Regulated Market Price](image-url)
In Figure 2-11, we graph the RM price by time of day (Hour 0 represents 12am) over all the data points, and observe a diurnal pattern of spot prices being highest in late afternoon and lowest in the middle of the night, likely due to higher demand in the afternoon and lower demand at night. Average prices for each hour of day are always greater than median prices because of spikes in spot prices as shown in the second graph, and to account for these infrequent but important extreme values, the simulation uses the average RM price by hour of day as shown in Figure 2-12.
Figure 2-12: Average Regulated Market Price by Hour of Day
2.3 Demand Data

An electric utility supplied to CASTLE Lab the demand data, which contains hourly demand for a set of transformers given in percent of transformer capacity.

Graphing data for a single transformer in Figure 2-13, demand appears to vary hourly but not be as volatile as RM prices or wind speed, and looking at a 10 day demand sample, there appears to be a diurnal pattern with lower demand at early morning hours and higher demand later in the day.

![Hourly Load Percentage of a Single Transformer over Entire Time Period](image1)

![Hourly Load Percentage of a Single Transformer over 10 Days](image2)

Figure 2-13: Hourly Load Percentage of a Single Transformer
Similar to what we do for the RM price data, we compile the load percentages by hour of day; however, due to the large volume of data, we perform much of the analysis by splitting the demand data into two parts. Looking at Figure 2-14 above, demand for Part 1 appears to be slightly higher than demand for Part 2, but the difference appears to be insignificant. Demand reaches its maximum in the afternoon when people arrive home from work, and reaches its minimum in the early morning hours when most people are asleep.
Figure 2-15: Average and Median Load Percentages of Part 1 and Part 2

Looking at the data for each part in Figure 2-15, the average demand by hour of day always exceeds the median demand because similar to RM prices, demand spikes (such as from air conditioning on a hot summer afternoon) cause the average to be higher than the median. The maximum demand for each hour of the day is shown in Figure 2-16, and interestingly, can be higher than 100% because the transformer’s installed capacity is only a recommended value, not an absolute limit for the demand a transformer can handle.
Another issue is whether to weight the percentages from the transformers by installed capacity or whether to treat each transformer as having equal capacity.

Figure 2-17 seems to show little difference, but we decided that weighting would be more accurate. The normalized load percentages presented in the graph are found by dividing by the maximum load percentage.
The final average hourly demand data used in the simulation is shown in Figure 2-18.
2.4 PHEV User Behavior Data

Lacking data for PHEV driver behavior, we decided to make up realistic profiles for the percentage of vehicles driving or plugged in at a given hour of day (Hour 0 represents 12am to 1am). Because weekend schedules tend to be irregular, only workdays are modeled, and the typical job is projected to last from 9am to 5pm. A drive is assumed to take 1 hour, whether for a commute to work or other activities.

2.4.1 Driving Patterns

We do not have precise data for the percentage of vehicles driving by hour of day, but we do know, according to the Department of Transportation, that a small fraction of vehicles (fewer than 20%) are on the road at any one time (Short and Denholm, 2006). Thus, we set the percentage of vehicles driving at rush hours (8am-10am and 5pm-7pm) to 20%. We assume about 10% of vehicles are driving in the middle of the day because of errands, family commitments, or other activities, and assume a minimum of 2% of vehicles driving in the middle of the night. The percentage of vehicles driving is assumed to change gradually between these 4 time periods (rush hour counts twice), and Figure 2-19 presents this data.
2.4.2 Plug in Patterns

The profiles shown in Figure 2-20 estimate hours at which the car battery would be available for charging and discharging. We assume that being able to plug in a vehicle at home is a prerequisite to purchasing a PHEV; thus, the vehicle battery is modeled as available when the driver is at home.

In Scenario 1, the vehicle is plugged in all the time, which is unrealistic because vehicles are obviously on the road part of the day. This case is mainly used to debug code for the simulation.

In Scenario 2, the PHEVs are plugged in whenever not driving, which is also improbable because there is unlikely to be a charging station wherever a vehicle stops. This is considered a best case scenario for battery availability.

In Scenario 3, the cars are only plugged in at home, which is also unlikely because most drivers would want to be able to plug in at work.
In Scenario 4 which is also shown in Figure 2-21, the vehicles are plugged in half the time when not at home or driving. This seems to be the most realistic scenario because outside of home, cars should have access to the grid at some locations (e.g., work, but not others (e.g., the supermarket). The percentage of vehicles plugged in at a given hour ranges from around 50% to 98%, this scenario is used for simulation analysis.

**Figure 2-20: Percentage of Vehicles Plugged into the Grid at Each Hour under the Scenarios**

![Hourly Percentage of Vehicles Plugged in](image)

**Figure 2-21: Percentage of Vehicles Plugged into the Grid at each Hour under Scenario 4**

![Percentage of Vehicles Plugged in by Hour of Day](image)
Chapter 3. Model

Consider a power system consisting of a wind farm as the primary generating mechanism and PHEV batteries as a storage system. The electric utility supplies electricity to end-use consumers (which include PHEVs) based on energy generated by the wind turbines, energy available in storage, and energy purchased from the RM, and a model is established that minimizes this cost of meeting demand.

3.1 Assumptions

Several assumptions are made in constructing this model, and they are listed below.

3.1.1 Constant Wind Speed throughout the Hour

The speed of wind is assumed to be constant throughout the hour, and is assumed to change instantaneously in the next hour.

In the same way, the power output of the wind farm, which directly depends on the wind speed, is assumed to be constant throughout the hour in which it is being generated. As the model transitions into a new hour, the power generated is assumed to change instantaneously to reflect the power generated in the new hour (Schoppe, 2010).
3.1.2 Regulated Market

Transactions made on the RM are instantaneous, and the RM price is assumed to be constant throughout the hour. The RM is assumed to have infinite supply; therefore, any amount of energy the electric utility needs to purchase on the RM can be accommodated, and transactions made on the RM have no effect on RM prices (Schoppe, 2010).

The model assumes no electricity sales from the utility to the RM. Thus, no arbitrage can occur by buying energy at a cheaper price from the wind farm to sell at a more expensive price on the RM.

3.1.3 PHEV Battery Operations

We assume that a single energy aggregator (the electric utility) makes the charging and discharging decisions; therefore, we only keep track of the aggregate battery reserve and storage decisions. The model assumes that each PHEV under the aggregator has the same maximum capacity, and that the aggregate battery reserve must maintain at least a minimum level in case a driver needs to use his or her PHEV.

The model assumes that the battery reserve of the fleet remains at a constant level throughout the hour. Battery charging and discharging is assumed to occur instantaneously during each hour, and the total battery reserve is updated at the beginning of the next hour.
3.1.4 Constant Generating and Storage Capacity

The number of wind turbines and the number of PHEVs are assumed to remain constant throughout the year.

3.1.5 Demand

At every hour, the utility must meet the electricity demand from non-vehicular sources through a combination of wind power, PHEV storage, and purchases from the RM. We assume flat electric rates paid by end-use consumers, so the model considers the cost of supplying electricity, not the revenue obtained by selling it. Furthermore, the cost of electricity distribution is assumed to be 0.
3.2 Problem Set-Up and Terminology

The model is constructed according to a five part modeling technique (Powell, 2007). The five element structure organizes the problem into state variables, decision variables, exogenous information, transition functions, and the objective function.

3.2.1 Parameters

For a given day $d$, the hours range from $h = 0$ to $h = 23$. Hour 0 of day $d + 1$ follows from hour 23 of day $d$.

$D =$ The set of days, $D = \{0, 1, 2, ..., 364\}$.

$H =$ The set of hours, $H = \{0, 1, 2, ..., 23\}$.

$\rho_R =$ Coefficient used to convert the generated electricity to potential energy in the storage (storage unit/electricity unit).

$\rho_E =$ Coefficient used to convert the potential energy in the storage to electricity (electricity unit/storage unit).

$\rho_R \rho_E \leq 1$ denotes the round trip efficiency through the storage system.

$1 - \rho_R \rho_E$ refers to the conversion loss from storage.

$N^{EV} =$ The total number of PHEVs under the energy aggregator.

$C^{max} =$ The battery capacity of a single PHEV (MWh).

$C^{min} =$ The minimum battery level a single PHEV must maintain (MWh).

$Z^{dlim} =$ The percentage of overall battery reserve that can be discharged (\%).
\( D_{\text{drain}} \) = The average percentage of battery drained from 1 km of driving (%/km)

\( K \) = The constant term relating the cube of wind speed to the energy produced by 1 wind turbine in an hour (MWh/(m\(^3\)/s\(^3\)))

\( N_{\text{turbine}} \) = The total number of turbines in the wind farm.

\( \{G_{dh}\}_{h \in H, d \in D} \) = The goal of the battery reserve level during hour \( h \) of day \( d \) (%)

\( \{p_{dh}^{\text{penalty}}\}_{h \in H, d \in D} \) = The penalty price of the battery reserve level being under the goal during hour \( h \) of day \( d \) ($/MWh)

\( Z_{\text{charge}} \) = The maximum percentage of the battery that can be charged each hour (%)

\( Z_{\text{discharge}} \) = The maximum percentage of the battery that can be discharged each hour (%)

\( \{A_{dh}^{\text{distance}}\}_{h \in H, d \in D} \) = The average distance a driving PHEV travels during hour \( h \) of day \( d \) (km).

### 3.2.2 State Variables

\( v_{dh} \) = The wind speed during hour \( h \) of day \( d \) (m/s).

\( B_{dh} \) = The total battery reserve of the fleet during hour \( h \) of day \( d \) (MWh).

\( Q_{dh}^{\text{plugged}} \) = The percentage of the PHEV fleet plugged into the grid during hour \( h \) of day \( d \) (%).

\( Q_{dh}^{\text{driving}} \) = The percentage of the PHEV fleet driving during hour \( h \) of day \( d \) (%).

\( p_{dh}^{\text{wind}} \) = The price of obtaining electricity from wind power during hour \( h \) of day \( d \) ($/MWh)
\( p_{dh}^{\text{grid}} \) = The price of obtaining electricity from the grid (or RM) during hour \( h \) of day \( d \) ($/MWh).

\( L_{dh} \) = The load (excluding PHEVs) during hour \( h \) of day \( d \) (MW).

\( S_{dh} \) = The state of the system during hour \( h \) of day \( d \).

\[ = (v_{dh}, B_{dh}, Q_{dh}^{\text{plugged}}, Q_{dh}^{\text{driving}}, P_{dh}^{\text{wind}}, P_{dh}^{\text{grid}}, L_{dh}) \]

### 3.2.3 Decision Variables

\( x_{dh}^{\text{store}} \) = The amount of energy to charge \((x_{dh}^{\text{store}} \geq 0)\) or discharge \((x_{dh}^{\text{store}} < 0)\) from the PHEV battery system during hour \( h \) of day \( d \) (MWh).

\( x_{dh}^{\text{wind}} \) = The amount of energy to purchase from the wind farm during hour \( h \) of day \( d \) (MWh).

### 3.2.4 Exogenous Information

New exogenous information arrives at the beginning of the next hour \( h + 1 \), and the changes apply instantaneously. Hour 24 of day \( d \) means hour 0 of day \( d + 1 \).

\( \hat{v}_{d,h+1} \) = The change in wind speed from hour \( h \) to hour \( h + 1 \) of day \( d \) (m/s).

\( \hat{Q}_{d,h+1}^{\text{plugged}} \) = The change in percentage of the PHEV fleet plugged in from hour \( h \) to hour \( h + 1 \) of day \( d \) (%).

\( \hat{Q}_{d,h+1}^{\text{driving}} \) = The change in percentage of the PHEV fleet driving from hour \( h \) to hour \( h + 1 \) of day \( d \) (%).
\[ \hat{p}_{d,h+1}^{\text{wind}} = \text{The change in price of obtaining electricity from wind power from hour } h \text{ to hour } h + 1 \text{ of day } d \ ($/\text{MWh}). \]

\[ \hat{p}_{d,h+1}^{\text{grid}} = \text{The change in price of obtaining electricity from the grid from hour } h \text{ to hour } h + 1 \text{ of day } d \ ($/\text{MWh}). \]

\[ \hat{L}_{d,h+1} = \text{The change in load (excluding PHEVs) from hour } h \text{ to hour } h + 1 \text{ of day } d \ (\text{MW}). \]

\[ \hat{W}_{d,h+1} = \text{The exogenous information from hour } h \text{ to hour } h + 1 \text{ of day } d. \]

\[ = (\hat{\theta}_{d,h+1}, \hat{q}_{d,h+1}^{\text{plugged}}, \hat{q}_{d,h+1}^{\text{driving}}, \hat{p}_{d,h+1}^{\text{wind}}, \hat{p}_{d,h+1}^{\text{grid}}, \hat{L}_{d,h+1}) \]

### 3.2.5 Transition Functions

\[ v_{d,h+1} = v_{dh} + \hat{\theta}_{d,h+1} \]

\[ Q_{d,h+1}^{\text{plugged}} = Q_{dh}^{\text{plugged}} + \hat{Q}_{d,h+1}^{\text{plugged}} \]

\[ Q_{d,h+1}^{\text{driving}} = Q_{dh}^{\text{driving}} + \hat{Q}_{d,h+1}^{\text{driving}} \]

\[ p_{d,h+1}^{\text{wind}} = p_{dh}^{\text{wind}} + \hat{p}_{d,h+1}^{\text{wind}} \]

\[ p_{d,h+1}^{\text{grid}} = p_{dh}^{\text{grid}} + \hat{p}_{d,h+1}^{\text{grid}} \]

\[ L_{d,h+1} = L_{dh} + \hat{L}_{d,h+1} \]

\[ B_{d,h+1} = B_{dh} - (N_{EV}^{\text{max}})(D_{\text{drain}}/A_{\text{distance}})Q_{dh}^{\text{driving}} \]

\[ + \left\{ \begin{array}{ll}
\frac{x_{\text{store}}^{\text{store}}}{\rho_E}, & \text{if } x_{\text{store}}^{\text{store}} < 0 \\
\frac{x_{\text{store}}^{\text{store}}}{\rho_R}, & \text{if } x_{\text{store}}^{\text{store}} \geq 0
\end{array} \right. \]
### 3.2.6 Objective Function

\( E_{dh}^{grid} \) = The electricity purchased from the grid during hour \( h \) of day \( d \) (MWh)

\[
E_{dh}^{grid} = \max(L_{dh} - x_{wind}^{dh} + x_{store}^{dh}, 0)
\]

\( E_{dh}^{penalty} \) = The energy penalized during hour \( h \) of day \( d \) (MWh)

\[
E_{dh}^{penalty} = \max(G_{dh}N^E C_{ma}^{max} - B_{dh}, 0)
\]

\( C_{dh} \) = The operating cost for hour \( h \) of day \( d \).

\[
C_{dh} = p_{wind}^{dh}x_{wind}^{dh} + p_{grid}^{dh}E_{dh}^{grid} + p_{penalty}^{dh}E_{dh}^{penalty}
\]

\[
C_{dh} = C_{dh}(S_{dh}, x_{dh}^{s}(S_{dh}))
\]

\( C_{total} \) = The total operating cost during the course of one year.

\[
C_{total} = \sum_{d=0}^{364} \sum_{h=0}^{23} C_{dh}(S_{dh}, x_{dh}^{s}(S_{dh}))
\]

The objective of the model is to find the optimal operation policy among the set of policies \( \Pi \) that satisfies the following equation:

\[
\min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{d=0}^{364} \sum_{h=0}^{23} C_{dh}(S_{dh}, X_{dh}^{\pi}(S_{dh})) \right\}
\]

where \( X_{dh}^{\pi}(S_{dh}) = x_{store}^{dh} \)

Subject to:

\( S_{d,h+1} = S^M(S_{dh}, X_{dh}^{\pi}(S_{dh}), \bar{W}_{d,h+1}) \)

where \( S^M(\ldots) \) is the evolution of the state variable due to exogenous information and decision variables.
3.2.7 Constraints

The constraints for this model are listed below.

1. **Battery level boundary constraints:**

   \[ N^{EV}C^{\text{min}} \leq B_{dh} \leq N^{EV}C^{\text{max}}, \quad \forall h \in H, d \in D \]

   The battery level of the fleet can never go below the minimum allowable level or above the maximum battery capacity of the fleet.

2. **Charging constraints:**

   \[ x_{\text{store}}^{\text{charge}} \leq \min \left( \frac{Z^{\text{charge}}(N^{EV}C^{\text{max}})Q_{\text{plugged}}}{\rho_R}, \frac{N^{EV}C^{\text{max}} - B_{dh}Q_{\text{plugged}}}{\rho_R} \right), \forall h \in H, d \in D \]

   Limit charging by the percentage of vehicles plugged into the grid, the maximum battery capacity of the fleet, the maximum rate of charge, and the current battery reserve.

3. **Discharge constraints:**

   \[ x_{\text{store}}^{\text{discharge}} \geq \max \left( -\rho_E Z^{\text{discharge}}(N^{EV}C^{\text{max}})Q_{\text{plugged}}, -\rho_E B_{dh}Q_{\text{plugged}} \right), \forall h \in H, d \in D \]

   Limit discharging by the percentage of vehicles plugged into the grid, the maximum battery capacity of the fleet, the maximum rate of discharge, and the current battery reserve.
4. **Discharge limit constraint:**

\[ x_{dh}^{\text{store}} \geq 0, \quad \text{if } B_{dh} < N_{ev} C_{\text{max}} (1 - Z_{\text{dim}}), \quad \forall h \in H, d \in D \]

Do not allow discharging if the current battery level is below the discharge limit.

5. **Wind power purchasing constraints:**

\[ f(v_{dh}) = \text{The electricity produced by 1 wind turbine during hour } h \text{ of day } d \text{ given wind speed } v_{dh} \text{ (MWh).} \]

\[
\begin{cases}
0, & \text{if } v_{dh} < 3 \\
K v_{dh}^3, & \text{if } 3 \leq v_{dh} < 15 \\
K (15^3), & \text{if } v_{dh} \geq 15
\end{cases}
\]

\[ E_{dh}^{\text{wind}} = \text{The electricity produced by wind farm during hour } h \text{ of day } d \text{ (MWh)} \]

\[ = N_{\text{turbine}} f(v_{dh}) \]

\[ 0 \leq x_{dh}^{\text{wind}} \leq E_{dh}^{\text{wind}}, \quad \forall h \in H, d \in D \]

The electric utility cannot buy more electricity from the wind farm than what is produced. In addition, the utility can purchase electricity from the wind farm, but not sell it.
3.3 Policies

In order to measure the impact of PHEVs on electric utility costs, this paper studies four policies for making storage decisions.

3.3.1 Policy 1: V2G Vehicles with Charge and Discharge

In this case, the utility, not the PHEV drivers, is assumed to manage the charging and discharging decisions. The utility can charge the vehicles when the wind farm’s output rises or when demand falls, and deliver electricity discharged from the vehicle batteries when wind farm output drops or when demand rises.

3.3.2 Policy 2: V2G Vehicles without Discharge

Here, the utility still controls when to charge the PHEVs and the rate of charge, but cannot extract energy from the PHEV batteries. The vehicles still provide demand response services by allowing the utility charge the PHEVs more at a favorable time (e.g., lower demand load) and charge less at an unfavorable time (e.g., higher demand load).

3.3.3 Policy 3: Vehicles without V2G

In this policy, the PHEVs do not communicate with the grid; thus, the electric utility has no control over storage decisions. We assume that the vehicles plug into the grid immediately after driving, and the utility must replace the battery energy lost from driving in the previous hour.
3.3.4 Policy 4: No PHEVs or EVs

This is not a realistic scenario; however, this policy serves as a benchmark for measuring the effects of PHEVs on the power system.
We use stochastic dynamic programming to create the simulation of the model, and add assumptions to make the problem computable and constraints to insure proper program behavior. We also provide values for the parameters in the model.

4.1 Stochastic Dynamic Programming Algorithm

Stochastic dynamic programming is used to model optimization problems that involve uncertainty, and the goal is to find some policy that maximizes or minimizes some function ("Stochastic Programming"). These dynamic programs are solved by going backward in time.

Let $T$ be the final time increment. We first initialize

$$V_{T+1}(S_{T+1}) = 0,$$
where $S_t$ is the state of the system at time $t$, and $V_t(\cdot)$ is the value of a particular state at time $t$.

We then solve

$$V_{dh}(S_{dh}) = \min_{x_{dh}} \{ C_{dh}(S_{dh}, x_{dh}) + \mathbb{E}[V_{d,h+1}(S_{d,h+1})] \},$$

$$\mathbb{E}[V_{d,h+1}(S_{d,h+1})] = \sum_{v \in V} P(v_{d,h+1} = v)V_{d,h+1}(S_{d,h+1}|v_{d,h+1} = v),$$

$$S_{d,h+1} = S^M(S_{dh}, x_{dh}, \mathcal{W}_{d,h+1}),$$

where $x_{dh}$ is the decision made at hour $h$ of day $d$, $v_{d,h+1}$ is the velocity of wind at hour $h$ of day $d$, $v$ represents a particular wind velocity value, $V$ represents the set of possible wind velocities, and the final equation is the transition function given in Section 3.2.6. This dynamic program is solved for each state and time period.
4.2 Additional Assumptions

The following assumptions simplify the problem so that the dynamic program can be solved within a reasonable amount of time.

4.2.1 Utility Owns Wind Farm

We assume that the electric utility owns the wind farm. Since the fixed cost of building the wind turbines has already been paid, we assume that the marginal price of electricity from wind $p_{dh}^{\text{wind}}$ is 0. Moreover, because wind energy is free, the optimal amount of energy to buy from the wind farm each hour $x_{dh}^{\text{wind}}$ is all the energy generated by wind in that hour $E_{dh}^{\text{wind}}$.

4.2.2 Only Randomness is Wind

We realized that even without price of electricity from wind, the state variable is still 6-dimensional (7 if time is included), and would be impractical to compute. Thus, we eliminate all the exogenous information except wind speed, which we update using the hour-ahead probabilistic model discussed in Section 2.1.2. The regulated market price, the load, the percentage of vehicles driving, and the percentage of vehicles plugged in are deterministic by hour of day, and are analyzed in Sections 2.2 to 2.4. In all, the dynamic program only has 3 states: time increment, wind speed, and battery level.
4.3 Additional Constraints

The constraints added during coding include:

1. **Regulated market purchase constraint:**

   \[
   x_{d_{h}}^{\text{store}} \leq \frac{\left(N^{EV}c^{\text{max}}(D_{\text{drain}}^{\text{distance}})Q_{\text{driving}}^{\text{driving}}\right)}{\rho_{R}},
   \]
   \[\text{if } P_{d_{h}}^{\text{grid}}E_{d_{h}}^{\text{grid}} > \max(P_{d_{h}}^{\text{grid}}L_{d_{h}}) \text{ or if } E_{d_{h}}^{\text{grid}} > \max(L_{d_{h}}), \quad \forall h \in H, d \in D \]

   In order to prevent extreme RM purchases, if the RM electricity cost with vehicles is higher than the maximum RM electricity cost without vehicles, or if the RM purchase quantity with vehicles is higher than the maximum RM purchase quantity without vehicles, do not charge more than what is required for driving. This constraint is necessary to keep RM purchases reasonable.

2. **Wasteful discharge constraint:**

   \[
   x_{d_{h}}^{\text{store}} \geq 0, \quad \text{if } E_{d_{h}}^{\text{wind}} > L_{d_{h}}, \quad \forall h \in H, d \in D
   \]

   Do not allow battery discharge if the energy from wind is enough to satisfy load because that extra electricity cannot be sold to the RM and would be wasted. This constraint seems obvious, but without it, the dynamic program occasionally makes this suboptimal decision.
4.4 Parameter Values

Due to computing time constraints, we use a time horizon of 30 days instead of the full year.

The number of PHEVs $N^{EV}$ is varied in the results section.

As for conversion efficiencies $\rho_R$ and $\rho_E$, a paper that also models PHEVs as grid resources assumes a charging efficiency of 90% and a discharging efficiency of 93% (Sioshansi and Denholm, 2009). We use 90% efficiencies as a benchmark although we vary this parameter in the results section.

From another source, we find that the Chevrolet Volt has a 16 kWh battery with an electric-only range of 25 to 50 miles while the Nissan Leaf is equipped with a 24 kWh battery with a 50 to 100 mile range (Wynne and Warburton, 2010). We assume that battery technology will continue to improve, and use a 25 kWh PHEV battery for $C^{max}$.

The same source assumes that the average electricity use of the vehicle fleet is .35 kWh/mile or .22 kWh/km (Wynne and Warburton, 2010), which implies a drainage of 1.367%/km for the Volt and .911%/km for the Leaf. Thus, we assume 1%/km for $D^{drain}$.

We also find that the average load of a PHEV is 1.4 kW (Wynne and Warburton, 2010), which implies that on average, 8.75% of the Volt battery and 5.83% of the Leaf battery can be charged each hour. However, we assume that technologies
are being developed to allow for much quicker charges and discharges, so we use 15% for $Z^{charge}$, and a lower value of 10% for $Z^{discharge}$.

For simplicity, we combine the minimum level and discharge limit such that $C^{min} = C^{max}(1 - Z^{\text{dislim}})$. Also, we assume that $A^{\text{distance}}_{dh}$ is 20km (12.5 miles) for all hours, as that seems like a reasonable distance for a commute.

Because of charging constraints, it may be difficult to fully recharge the battery, so we assume a goal level $G_{dh}$ of 90% for all hours. The penalty price $p^{\text{penalty}}_{dh}$ is set to $1/MWh$ as a baseline although this is also varied in the results section. The only exception is that the penalty is 1000 times more expensive at 5 am in order to insure enough battery reserve for the morning commute. Another source assumes a 4 am constraint, but we believe that time may be unnecessarily early (Sioshansi and Denholm, 2009).

We coordinate the number of turbines $N^{turbine}$ and the non-vehicular demand, so that the average power from wind, which is calculated by multiplying the number of turbines by the installed capacity of each turbine (4 MW) and by the capacity factor of South New Jersey (8.76%), is about 20% of demand. Moreover, as computed in Section 2.1.1, $K = 1.185 * 10^{-3} \frac{MWh}{(m/s)^3}$.

Finally, we assume a 5 m/s initial wind speed because the median wind speed of South New Jersey is 5.15 m/s as discussed in Section 2.1.2, and assume an initial battery level of 95%.
Chapter 5. Results of Simulation

This chapter presents a summary of results from the simulations. We display the parameters used and statistics collected, show a sample path of the simulation, and demonstrate the effects of changing the number of vehicles, the discharge limit, the conversion efficiencies, and the battery penalty.

5.1 Simulation Inputs and Outputs

Table 5-1 gives the list of parameters used as input for a particular simulation. The bolded parameters are varied in this results chapter while the other parameters are fixed.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Days</td>
<td>30</td>
</tr>
<tr>
<td>Number of Turbines</td>
<td>200</td>
</tr>
<tr>
<td><strong>Number of Vehicles</strong></td>
<td><strong>50000</strong></td>
</tr>
<tr>
<td>Total Battery Capacity (MWh)</td>
<td>1250</td>
</tr>
<tr>
<td>Maximum Non-vehicular Load (MW)</td>
<td>350</td>
</tr>
<tr>
<td>Average Wind Power as a Percentage of Maximum Non-vehicular Load (%)</td>
<td>20.02</td>
</tr>
<tr>
<td><strong>Maximum Driving Load from 1 hour as a Percentage of Maximum Non-vehicular Load (%)</strong></td>
<td><strong>14.29</strong></td>
</tr>
<tr>
<td>Charge Efficiency (%)</td>
<td>95</td>
</tr>
<tr>
<td>Discharge Efficiency (%)</td>
<td>95</td>
</tr>
<tr>
<td>Maximum Charge Rate (%/hr)</td>
<td>15</td>
</tr>
<tr>
<td>Maximum Discharge Rate (%/hr)</td>
<td>10</td>
</tr>
<tr>
<td>Initial Battery Level (%)</td>
<td>95</td>
</tr>
<tr>
<td>Initial Wind Speed (m/s)</td>
<td>5</td>
</tr>
<tr>
<td><strong>Discharge Limit (%)</strong></td>
<td><strong>50</strong></td>
</tr>
<tr>
<td>Vehicle Plug-in Scenario</td>
<td>4</td>
</tr>
<tr>
<td>Average Distance Driven in an Hour (km)</td>
<td>20</td>
</tr>
<tr>
<td>Average Drainage of Battery (%/km)</td>
<td>1</td>
</tr>
<tr>
<td>Battery Level Goal (%)</td>
<td>90</td>
</tr>
<tr>
<td><strong>Penalty Cost for not Meeting Battery Level Goal ($/MWh)</strong></td>
<td><strong>1</strong></td>
</tr>
</tbody>
</table>
Table 5-2: Statistics Collected from a Simulation.

1000 Sample Paths:

<table>
<thead>
<tr>
<th>Policy</th>
<th>V2G Cars with Charge and Discharge</th>
<th>V2G Cars without Discharge</th>
<th>Cars without V2G</th>
<th>No Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Total Energy (MWh)</td>
<td>193,362.06</td>
<td>192,963.24</td>
<td>192,781.80</td>
<td>175,599.43</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>10,280,086.82</td>
<td>10,472,603.03</td>
<td>10,765,084.83</td>
<td>9,728,811.11</td>
</tr>
<tr>
<td>Average 80th Percentile Energy (MWh)</td>
<td>344.85</td>
<td>336.22</td>
<td>350.69</td>
<td>316.21</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>19,728.79</td>
<td>20,799.82</td>
<td>22,164.94</td>
<td>20,025.00</td>
</tr>
<tr>
<td>Average 90th Percentile Energy (MWh)</td>
<td>349.16</td>
<td>346.82</td>
<td>365.45</td>
<td>330.98</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>21,229.74</td>
<td>21,921.86</td>
<td>23,657.77</td>
<td>21,369.45</td>
</tr>
<tr>
<td>Average 95th Percentile Energy (MWh)</td>
<td>351.70</td>
<td>351.33</td>
<td>372.13</td>
<td>338.24</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>21,960.53</td>
<td>22,505.44</td>
<td>24,813.40</td>
<td>22,242.47</td>
</tr>
<tr>
<td>Average Worst Case Energy (MWh)</td>
<td>369.19</td>
<td>367.68</td>
<td>386.24</td>
<td>349.89</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>22,848.35</td>
<td>24,676.24</td>
<td>27,385.88</td>
<td>24,676.24</td>
</tr>
<tr>
<td>Maximum Worst Case Energy (MWh)</td>
<td>374.10</td>
<td>374.10</td>
<td>386.41</td>
<td>350.00</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>24,292.69</td>
<td>24,697.85</td>
<td>27,407.49</td>
<td>24,697.85</td>
</tr>
<tr>
<td>Average Total Percent Below Goal Level</td>
<td>8,279.39</td>
<td>4,560.41</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

For a simulation, we use 1000 sample paths and collect statistics for each policy as shown in Table 5-2.

We first record the total amount and cost of RM electricity purchased for each sample path, and average these numbers over the 1000 paths. We do not consider the cost of not meeting the battery goal because the battery penalty is meant to encourage favorable behavior from the dynamic program (i.e., do not leave battery level far below the goal all the time) and not meant to compare policies.
As a general observation, we find that Policy 4 consistently has the lowest average total RM purchases because there is no additional cost from meeting vehicle load. Policy 3 has the highest average total RM cost because the utility must charge the vehicles after they are driven and cannot choose to buy more energy from the RM when prices are lower. Policy 2 has a lower RM cost than Policy 3 because the electric utility is able to buy less on the RM when prices are higher and buy more when prices are lower. Policy 1 is even better than Policy 2 because RM purchases can be further reduced when prices are high (by discharging vehicle batteries).

The output table also contains the total percent below the goal level over time averaged over the sample paths, and we can use this as a measure of how much storage the vehicle batteries provide. This number is always 0 for the last two policies because there are either no vehicles (Policy 4) or vehicles are charged immediately after being driven so battery level does not fall below the 90% goal (Policy 3). Based on this number, Policy 1 always uses the batteries more intensively than Policy 2, which provides a higher storage benefit as observed above, but also results in slightly more energy being purchased from the RM due to conversion losses.

We also record the average 80\textsuperscript{th} percentile, the average 90\textsuperscript{th} percentile, the average 95\textsuperscript{th} percentile, and the average and maximum worst case (i.e., highest single time period) RM purchase quantity and cost in order to capture the more extreme RM purchase events. In results analysis, however, we use average total statistics rather than these statistics measuring extremes because we are more concerned with the
overall performance of the policies. Nonetheless, these statistics are useful in
providing the insight that adding vehicles may exacerbate worst case RM purchases.

If wind speed is low for an extended period, the battery drains from driving and can no
longer be discharged, and required recharging increases RM purchases even more.

In this chapter, we focus our attention on Policies 1 and 2, specifically on the
difference in average total cost as that represents the benefit of being able to discharge
the PHEV. In this particular simulation, the marginal benefit of Policy 1 is

$192,516.21 ($10,472,603.03-$10,280,086.82).
5.2 Sample Path of a Simulation

For the simulation represented by Tables 5-1 and 5-2, we analyze one of the 1000 sample paths.

From the plots in Figure 5-1, we see that wind speed is highly volatile, but tends to be close to the previous hour’s wind speed, as would be expected under the probabilistic model. In both 30 day and 5 day time horizons, the wind speed appears to fluctuate around 5 m/s.

![Simulated Wind Speed over 30 Days](image1)

![Simulated Wind Speed over 5 Days](image2)

**Figure 5-1: Hourly Wind Speed**
The storage decisions and corresponding battery levels for the first 5 days are given in Figure 5-2, and we observe that under both policies, the battery is nearly full early in the day, but falls as the day progresses. During daytime, driving drains the batteries, but significant charging does not occur as RM prices are more expensive. Indeed, under Policy 1, the battery level drops more heavily (to nearly 50% at the end of the day) because battery energy is lost from both driving and discharges by the utility. For both policies, large amounts of charging occur in the middle of the night because RM prices are cheaper and the 5am battery constraint must be met. The lower RM prices appear to be significant because the battery is charged above the 90% level that the 5am constraint requires.
Averaging the decisions by hour of day in Figure 5-3, we see that Policy 1 is able to discharge in late afternoon, but must charge more than Policy 2 does in early morning to compensate. In the second plot, we normalize hourly RM prices (maximum price being 1 and minimum price being -1), and we normalize the highest magnitude average decision to be 1. We note that both policies charge the most when RM prices are lowest, and when prices are highest, Policy 2 does not charge at all and Policy 1 discharges. This demonstrates graphically that the discharge ability of Policy 1 enables it to make fewer RM purchases at higher prices relative to Policy 2, which explains why Policy 1 does better.
Indeed, as shown in Figure 5-4, Policy 1 purchases a relatively higher quantity of RM electricity in the middle of the night while Policy 2 purchases relatively more energy from the RM at late afternoon. From the second plot, the additional cost of RM electricity at night under Policy 1 appears to be less than the additional cost of RM electricity in the afternoon under Policy 2, and these differences each day add up to form the marginal benefit of Policy 1.
Considering RM purchases from all policies in Figure 5-5, we see that Policies 2 and 3 appear to always purchase more RM electricity than Policy 4 because of their inability to discharge. Relative to Policy 4, Policy 1 tends to purchase slightly less in the afternoon but far more in the middle of the night, and the additional energy Policy 1 must purchase to satisfy vehicular load results in a higher total cost than Policy 4.

Figure 5-5: Hourly RM Purchases under All Policies
Figure 5-6 shows the effect of PHEVs on load under Policy 1. As shown above, increased charging at night causes total load to increase when it is cheaper to meet (lower RM prices), and discharging in the afternoon causes total load to decrease when it is more expensive to meet (higher RM prices). This charging and discharging pattern appears to repeat every day.

When wind output rises, wind energy can supply a greater share of load, so fewer RM purchases are needed to meet non-vehicular demand. The utility is willing to charge the vehicles more in the night and late morning because the RM cost is cheaper (due to lower quantity of RM electricity needed). The electric utility is also able to discharge more in the afternoon because it charged the batteries more during late morning. In the rare case where wind output rises above non-vehicular load, the utility is able to use the excess wind energy to charge the batteries.
5.3 Varying Number of Vehicles

In Figure 5-7 below, we measure the marginal benefit of Policy 1 over Policy 2 varying over the number of PHEVs. As for the other parameters, we have 30% discharge limit, 90% charge and discharge efficiencies, and $1/MWh battery penalty.

Initially, marginal benefit steadily increases as the number of vehicles rises because more vehicles provide greater storage capacity which leads to higher benefit from discharging. However, there are diminishing returns above 40000 vehicles because discharging more energy in the afternoon would require charging more energy at night, but that is limited by the regulated market purchase constraint discussed in Section 4.3. The graph also implies that under these parameters, 40000 or more vehicles are necessary to provide optimal balancing regulation for 200 wind turbines.

We also note that under these parameters, the marginal benefit is quite small, and taking the point where the curve appears to bend (40000 PHEVs and $6000 benefit), we find a discharge benefit of only $.15/vehicle over the 1 month horizon. Intuitively, this quantity is too small to convince PHEV drivers to allow discharging of up to 30% of their vehicles batteries. In the rest of this chapter, we modify some of the parameters to see if a more economically feasible deal exists.
Figure 5-7: Varying Number of Vehicles
5.4 Varying Discharge Limit

We next change the discharge limit as shown in Figure 5-8 while maintaining the 90% conversion efficiencies and $1/MWh battery penalty. As seen in both the 10000 vehicle and 50000 vehicle cases, marginal benefit rises dramatically with a higher discharge limit because each vehicle has a higher storage capacity. There are diminishing returns above a certain discharge limit because a deeper discharge would require a deeper charge later and no more than 15% of the battery can be charged in an hour.

At a 50% discharge limit, the marginal benefit is $2/car in the 50000 vehicle case and $3/car in the 10000 vehicle case. This is still a small amount, but discharging now makes more economic sense than it did with a 30% discharge limit.
Figure 5-8: Varying Discharge Limit
5.5 Varying Conversion Efficiencies

Interestingly, the marginal benefit curve in Figure 5-8 is S-shaped, as there is very little marginal benefit at lower discharge limits, and we decided to investigate this by varying charge and discharge efficiencies as seen in Figure 5-9. We keep the $1/MWh battery penalty, and select the 50000 car case to obtain larger numbers.

As efficiency rises, the marginal reward increases by a sizable amount. In addition, while at 85% and 90% efficiencies the marginal benefit leaves the bottom of the S-shaped curve at 40% discharge limit, the marginal benefit takes off at a 30% limit for 95% efficiency, and there is no S-curve effect at 100% efficiency.

The explanation lies in a cost-benefit analysis. By being able to discharge, there is the benefit of buying less RM electricity at higher prices (discharge at those times), but there is the cost of energy conversion loss. With 100% conversion efficiency, there is no cost, so there is much higher net benefit from being able to discharge the battery. However, at lower conversion efficiency (like 90%), the costs are higher, so discharging is only useful if benefit is higher (more discharging allowed by higher allowable fraction of the battery, like over 30%).

These results imply that improving efficiency has important economic consequences. For instance, given a 50% discharge limit, raising the charge and discharge efficiencies from 90% to 95% would increase marginal benefit from $2/car to $4/car.
Figure 5-9: Varying Conversion Efficiencies
5.6 Varying Battery Penalty

In this section, we observe the effect of varying the penalty cost for not meeting battery goal.

According to Figure 5-10, there appears to be an optimal penalty cost around $3/MWh. At smaller penalties, the battery remains at a low level because there is little incentive to charge, which means less battery energy is available when discharging is needed more (i.e., low wind output and higher RM prices). The marginal benefit also falls under larger penalties because not much discharging occurs.

There appear to be huge economic benefits from tuning this penalty parameter, as changing the penalty cost from $1/MWh to $3/MWh causes marginal benefit to increase from around $.10/car to $1/car.

![Figure 5-10: Varying Battery Penalty](image)
Using the new battery penalty of $3/MWh, we again vary the discharge limit and conversion efficiency. From Figure 5-11, using $3/MWh appears to give similar results as $1/MWh. The only major difference is that marginal benefits for the 30% discharge limit seem much greater.
We decided to do some direct graphical comparisons between $3/MWh and $1/MWh. From varying the number of vehicles in Figure 5-12, it appears that a penalty of $3/MWh performs much better than a penalty of $1/MWh. Under this set of parameters, 60000 cars provide optimal balancing regulation for the 200 wind turbines.

Interestingly, the marginal benefit decreases after a certain number of vehicles (60000 in this case). Because the penalty is in units of $/MWh rather than $/(% below goal), the penalty has a greater effect in the case of more vehicles, which inhibits discharging.

![Figure 5-12: Varying Number of Vehicles for the Two Battery Penalties](image)
However, at other discharge limits, the advantage of $3/MWh over $1/MWh disappears. As shown in Figure 5-13 below, for the different conversion efficiencies, $3/MWh does better at 30% discharge limit, but worse at higher discharge limits likely because the higher penalty discourages deeper discharges. At higher conversion efficiencies though, the difference in reward between the battery penalties appears be less probably because the benefit from discharging is so large (due to no conversion losses) that this higher battery penalty does not change discharging much. These results imply that there is an optimal battery penalty for each discharge limit and possibly also for each conversion efficiency.
Figure 5-13: Varying Discharge Limit for the Two Battery Penalties
Chapter 6. Conclusions and Future Research

As wind energy provides a greater share of the nation’s energy needs, the volatility of wind will be its most serious problem. The best solution is to use a cheap, efficient, and readily available form of storage. This thesis studies the possibility of using plug-in hybrid electric vehicles as that type of storage, potentially yielding benefits to the electric utility, wind farm operator, and PHEV driver and aiding the expansion of both PHEVs and wind energy.

In developing the simulation of the effects of PHEVs on electric utility costs, we consider the randomness of wind, but treat price of electricity on the regulated market, demand, percentage of vehicles plugged in, and percentage of vehicles driving as deterministic by hour of day. In future research, these factors would be modeled with uncertainty, and other sources of randomness like seasonality would be considered. Nonetheless, this assumption does not appear to negatively impact the results, and is a reasonable one to make, as these factors do have regular daily patterns.

In modeling the hourly wind, we may overestimate the volatility of wind energy because we use only one wind farm and assume that each turbine gives the same output. Further research might use multiple wind farms that are not perfectly correlated, or perhaps choose a location with more reliable winds. Nevertheless, our simulation still obtains the crucial result that PHEVs can offset this wind volatility.
In making assumptions, the model may oversimplify the electricity market. Future research may consider the scenario where generator and retailer are different companies such that wind energy is no longer free, and may also study the day ahead market for electricity. Other energy sources could be included in the model to give a better estimate of cost.

If the model presented in this paper is studied in greater depth, several changes could be made. A researcher could include a battery level constraint for the afternoon (possibly around 3pm) similar to the one used for 5 am in order to provide a full battery to drivers commuting home from work. Future research could also add a nonlinear cost function for battery penalty in order to discourage deep discharges. In addition, the battery penalty could be fine-tuned in order to find the optimal benefit for a given set of parameters. Finally, more time could be spent tuning parameters in order to find more profitable deals for the electric utility and PHEV drivers.

The thesis models the combination of PHEVs and wind energy with some uncertainty including random wind speed and storage decisions that depend on the simulation state. It is this better understanding of how to model and simulate the problem and a few insights gained from the results that are contributed to the academic community.
The MATLAB program presented here, windcarDP.m, represents both the model from Chapter 4 and 5 and the simulation from Chapter 6. The code solves the dynamic program for a given set of inputs, and then runs a forward simulation using 1000 sample paths. The code below contains a sample set of inputs.

```matlab
%Inputs
%Unchangeable Inputs
%number of time increments where decisions are made (in hours)
H=24;

%maximum wind speed (in m/s)
Vmax=20;

%Tunable Parameters
%number of days
D=30;

%number of wind turbines
NumTurb=200;

%number of vehicles
Nveh=50000;

%battery capacity per car (in MWh)
CarCap=.025;

%load multiplier (in MW)
LMult=350;

%price of electricity from wind (in $/MWh)
Pwind=0;

%charge efficiency (in proportion)
ChE=.85;
```
% discharge efficiency (in proportion)
DisE = .85;

% part of battery that can be charged each hour (in proportion)
chrate = .15;

% part of battery that can be discharged each hour (in proportion)
disrate = .10;

% initial battery level (in proportion)
Binitial = .95;

% discharge limit (in proportion)
DLimProp = .10;

% scenario of vehicles plugged into grid
S = 4;

% average number of km driven in an hour (in km)
Avdrive = 20;

% average drainage of battery from a km of driving (in proportion)
Avdrain = .01;

% penalty cost for not meeting battery level goal by hour (in $/MWh)
PCost = 3;

% penalty cost for buying more energy from grid than maximum load (in $/MW)
PLCost = 0;

% penalty cost for paying more for grid energy than the maximum cost
% without cars
PCCost = 0;

% average wind speed (in m/s)
Avwind = 5;

% Scaling

% number of time increments where decisions are made (in hours)
T = H*D;

% total amount of battery capacity (in MWh)
BatCap = Nveh*CarCap;

% scale for battery reserve of fleet (in tenth of a percent)
Bscale = 1000;
%make decision scale for charging (in percent)
Dchsc=chrate*Bscale;

%make decision scale for discharging (in percent)
Ddissc=disrate*Bscale;

%amount to iterate decision by (in tenth of a percent)
Diter=1;

%number of possible decisions
Dpos=(chrate+disrate)*100+1;

%convert initial battery level to scale (in tenth of percent)
Bi=Binitial*Bscale;

%minimum battery level where discharging is allowed (in proportion)
Bminimum=1-DLimProp;

%convert minimum battery level to scale (in tenth of percent)
Bmin=floor(Bminimum*Bscale);

%Lookup Tables

%power of one Wind turbine for given Wind speed (in MW)
Pturb=zeros(Vmax+1,1);
for i=1:(Vmax+1)
    %generate energy from power only if wind speed is greater than or
    %equal to 3 m/s
    if i<4
        Pturb(i)=0;
    elseif (i>16)&&(i<=25)
        Pturb(i)=1185*15^3;
    elseif i>25
        Pturb(i)=0;
    else
        %formula for wind speeds from 3 m/s to 15 m/s
        Pturb(i)=1185*(i-1)^3;
    end
end
Pturb=Pturb/10^6; %convert W to MW

%find wind speed transition probabilities
Windtransitions=[89, 235, 32, 16, 16, 22, 11, 16, 9, 3, 5, 3, 0, 3, 0, 0, 0, 0, 0, 0, 0; 231, 1662, 900, 55, 5, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; 36, 905, 3762, 1454, 79, 4, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; 16, 49, 1480, 7126, 1852, 98, 12, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0; 15, 2, 63, 1919, 8145, 1978, 75, 5, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0; 21, 1, 2, 55, 2051, 8302, 1873, 93, 8, 1, 1, 0, 0, 0, 0, 0, 0, 0; 16, 1, 2, 7, 49, 1950, 7167, 1527, 70, 6, 1, 2, 1, 0, 0, 0, 0, 0; 10, 0, 0, 3, 4, 44, 1606, 5274, 1147, 72, 2, 1, 1, 0, 0, 0, 0, 0; 9, 0, 1, 1, 1, 4, 48, 1197, 3835, 855, 49, 8, 2, 1, 0, 0, 0, 0, 0; 5, 0, 0, 0, 0, 1, 3, 2, 43, 908, 2762, 603, 38, 6, 0, 1, 0, 0, 0, 0, 0; 5, 0, 0, 0, 0, 0, 1, 5, 29, 634, 1874, 417, 17, 6, 0, 0, 0, 0, 0; 3, 0, 0, 0, 0, 0, 0, 0, 2, 2, 4, 33, 439, 1103, 234, 17, 2, 0, 0, 0, 0; 3, 0, 0, 0, 0, 0, 0, 0, 4, 14, 256, 667, 137, 9, 0, 1, 0, 0, 0; 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 8, 158, 389, 76, 3, 2, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 4, 79, 151, 32, 5, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1;].
RowSum=zeros(Vmax+1,1);
for i=1:(Vmax+1)
    for j=1:(Vmax+1)
        RowSum(i)=RowSum(i)+Windtransitions(i,j);
    end
end
Windprob=zeros(Vmax+1,Vmax+1);
for i=1:(Vmax+1)
    for j=1:(Vmax+1)
        Windprob(i,j)=Windtransitions(i,j)/RowSum(i);
    end
end

%goal for battery level by hour (in proportion)
G=[0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9, 0.9];

%relative penalty cost for not meeting battery level goal by hour (in proportion)
PenCost=[1, 1, 1, 1, 1, 1000, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1];
PenCost=PCost*PenCost;
% penalty cost in terms of battery units (in $ per battery unit)
a = PenCost*BatCap/Bscale;

% penalty costs for not meeting battery level goals (in $)
Penalty = zeros(T+1, Bscale+1);
for t=1:1:(T+1)
    h = rem(t-1, H)+1;
    for r=0:1:Bscale
        Penalty(t, r+1) = a(h) * max(G(h)*Bscale-r, 0);
    end
end

% load proportion by hour of day (in proportion)
% maximum load hour has value of 1
Loadprop = [0.806394093; 0.758420453; 0.72560071; 0.70134001; 0.69136659; 0.699134817; 0.727613019; 0.784948674; 0.839739741; 0.882329553; 0.92690355; 0.962486496; 0.978861628; 0.990295354; 0.998678473; 1; 0.994994194; 0.976587051; 0.961175693; 0.960240144; 0.961033852; 0.954087278; 0.920383463; 0.86872596];

% load by hour of day (in MW)
Load = LMult*Loadprop;
%find peak demand
maxL=max(Load);

%price of electricity on the grid market (in $/MWh)
Pricegrid=[38.52233855; 37.79106195; 34.97381488; 33.07377033; 34.06542234; 38.81943409; 51.58199467; 53.79905902; 52.87606697; 56.72802335; 61.86863423; 62.00314426; 61.4863177; 62.39512189; 62.6001466; 63.03268528; 67.33979407; 72.25704229; 67.3342558; 64.6401039; 67.61505906; 59.84831138; 45.7163771; 40.88592737];

%find peak price
maxPgrid=max(Pricegrid);

%find peak cost of purchasing entire load from grid
Gridcost=Load.*Pricegrid;
maxGcost=max(Gridcost);

%vehicles plugged into the grid (in proportion)
each column is a scenario
Pluggedveh=[1, 0.98, 0.98, 0.98; 1, 0.98, 0.98, 0.98; 1, 0.98, 0.98, 0.98; 1, 0.98, 0.98, 0.98];
%proportion of vehicles driving (in proportion)
Driving=[0.02;
0.02;
0.02;
0.02;
0.03;
0.05;
0.1;
0.15;
0.2;
0.2;
0.15;
0.1;
0.1;
0.1;
0.1;
0.1;
0.15;
0.2;
%changes in aggregate battery level from driving (in proportion)
Drivingimpactprop=Avdrive*Avdrain*Driving;

%scale changes in aggregate battery level from driving (in tenth of percent)
Drivingimpact=Drivingimpactprop*Bscale;

%find loads from driving assuming no additional charging or discharging (in MW)
DriveLoad=(Drivingimpact/Bscale)*BatCap;

%find total load assuming no additional charging or discharging (in MW)
TotLoad=Load+DriveLoad;

%--------------------------------------------------------------------

%Setup

%Setup for dynamic program
%cost matrix (in $)
Cost=zeros(T,Bscale+1,Vmax+1);

%decision matrix (in tenth of percent)
X=zeros(T,Bscale+1,Vmax+1);

%decision matrix of no discharging (in tenth of percent)
XND=zeros(T,Bscale+1,Vmax+1);

%value matrix (in $)
Value=zeros(T+1,Bscale+1,Vmax+1);

%Setup for simulation
%matrix with simulated costs (in $)
SimCost=zeros(T,1);
%matrix with aggregate simulated costs (in $)
SimTC=zeros(T,1);

%matrix with simulated battery reserves (in tenth of percent)
SimBat=zeros(T+1,1);

%matrix with simulated battery reserves if no discharging allowed (in %tenth of percent)
SimBatD=zeros(T,1);

%matrix with simulated total charges or discharges
SimSmC=zeros(T+1,1);

%matrix with simulated decisions (in tenth of percent)
SimX=zeros(T,1);

%matrix with simulated energy from decisions (in MWh)
SimEX=zeros(T,1);

%matrix with simulated decisions with no discharging (in tenth of percent)
SimXND=zeros(T,1);

%matrix with simulated wind speeds (in m/s)
SimWS=zeros(T,1);

%matrix with simulated energy from wind (in MWh)
SimEW=zeros(T,1);

%matrix with simulated costs of purchasing energy from wind (in $)
SimCW=zeros(T,1);

%matrix with simulated energy from grid (in MWh)
SimEG=zeros(T,1);

%matrix with simulated costs of purchasing energy from grid (in $)
SimCG=zeros(T,1);

%matrix with simulated penalty costs (in $)
SimCP=zeros(T,1);

%matrix with simulated energy from wind and cars (in MWh)
SimEWX=zeros(T,1);
%matrix with simulated energy drained from driving (in MWh)
SimED=zeros(T,1);

%matrix with load from non-vehicular sources at each hour (in MW)
SimL=zeros(T,1);

%matrix with load from all sources at each hour (in MW)
SimLX=zeros(T,1);

%matrix with energy from average wind speed of 5 m/s (in MWh)
SimEWA=zeros(T,1);

%find energy from grid if there were no cars (in MWh)
SimEGN=zeros(T,1);

%find cost of energy from grid if there were no cars (in $)
SimCGN=zeros(T,1);

%find energy from grid if cars are charged immediately after being
%driven (in MWh)
SimEGC=zeros(T,1);

%find cost of energy from grid if cars are charged immediately after
%being driven (in $)
SimCGC=zeros(T,1);

%find energy from grid if no discharging allowed (in MWh)
SimEGD=zeros(T,1);

%find cost of energy from grid if no discharging allowed (in $)
SimCGD=zeros(T,1);

%find percent below goal level each time period (in percent)
SimPBG=zeros(T,1);

%find percent below goal level if no discharging allowed (in percent)
SimPBGD=zeros(T,1);

%---------------------------------------------------------------
% Solve Dynamic Program for each state and time period

%Initialize Dynamic Program
%decision assumed to be 0 at time T+1
%find hour of time T+1
h=rem(T, H)+1;

%load at time T+1 (in MW)
L=Load(h);

%grid price at time T+1 (in $/MWh)
Pgrid=Pricegrid(h);

for r=Bmin:1:Bscale
    %index for current battery reserve
    rcurr=r+1;
    %look up penalty at this battery level (in $)
    y=Penalty(T+1, rcurr);

    for v=0:1:Vmax
        %index for current wind speed
        vcurr=v+1;

        %calculate total power corresponding to wind speed (in MW)
        Power=Pturb(vcurr)*NumTurb;

        %calculate value at time T+1
        Value(T+1, rcurr, vcurr)=Pwind*Power+Pgrid*(L-Power)+y;
    end
end

%Backwards Dynamic Program
for t=T:-1:1
    %hour corresponding to time t
    h=rem(t-1, H)+1;

    %load at time t (in MW)
    L=Load(h);

    %grid price at time t (in $/MWh)
    Pgrid=Pricegrid(h);

    %proportion of vehicles plugged in at time t
Bveh=Pluggedveh(h, S);

% limit magnitude of discharging decisions by percentage of vehicles
% plugged in (in tenth of percent)
Dmagdis=floor(DisE*Ddissc*Bveh);

% changes in aggregate battery level from driving at time t (in % tenth of percent)
DI=Drivingimpact(h);

% energy equivalent of driving impact (in MWh)
DriveE=(DI/Bscale)*BatCap;

for r=Bmin:1:Bscale
    % index for current battery reserve
    rcurr=r+1;

    % limit magnitude of charging decisions by percentage of vehicles plugged in and by current battery level (in tenth of percent)
    Dmagch=floor(min(Dchsc*Bveh/ChE, (Bscale-r)*Bveh/ChE));

    % look up penalty at this battery level (in $)
y=Penalty(t, rcurr);

    for v=0:1:Vmax
        % index for current wind speed
        vcurr=v+1;

        % calculate total power corresponding to Wind speed (in MW)
        Power=Pturb(vcurr)*NumTurb;

        % keep track of minimum value
        minval=Inf;

        % keep track of decision giving minimum value
        minx=0;

        % keep track of minimum value for no discharging
        minvalxnd=Inf;

        % keep track of decision giving minimum value for no discharging
        minxnd=0;

        % other code
    end
end
% keep track of minimum cost associated with minimum value
mincost = 0;

% keep track of value of each decision
val = 0;

% keep track of cost of each decision
cost = 0;

% keep track of value with no discharging of each decision
valxnd = 0;

% iterate decision by percent
for x = -Dmagdis : Diter : Dmagch
    % convert decision into energy (MWh)
    DecE = (x / Bscale) * BatCap;

    % power obtained from grid
    Powergrid = L - Power + DecE;

    % cost of power obtained from grid
    GCost = Pgrid * Powergrid;

    % cost given state and decision
    % allow purchases from grid but not sales to grid
    if (Powergrid > 0)
        cost = Pwind * Power + GCost + y;
    else
        cost = Pwind * Power + y;
    end

    % index for battery reserve in next time period
    if (x > 0)
        rnext = r + ChE * x - DI + 1;
        roundrnext = round(rnext);
    else
        rnext = r + x / DisE - DI + 1;
        roundrnext = round(rnext);
    end

    % decision constraints
    % do not allow decisions that cause next period battery
    % reserve to be beyond maximum or below minimum
    if (rnext > (Bscale + 1))
        cost = Inf;
rnext=Bscale+1;
roundrnext=rnext;
elseif (rnext<(Bmin+1))
cost=Inf;
rnext=Bmin+1;
roundrnext=rnext;
end

%forbid discharging if current battery level is
%below minimum battery requirement
if ((x<0)&&(r<Bmin))
cost=Inf;
end

%limit discharge by percentage of vehicles plugged in
if (x<(-Bveh*r*DisE))
cost=Inf;
end

%do not allow battery level to go below minimum if
%present level is at or above minimum
if ((r>=Bmin)&&(rnext-1)<Bmin)
cost=Inf;
end

%do not allow charging more than what is needed for
%driving if cost of energy bought from grid is
greater
%than peak cost with no charging
if (((GCost>maxGcost)&&(x>(DI/ChE+1)))||(Powergrid>maxL)&&(x>(DI/ChE+1))
)  
cost=Inf;
end

%penalize buying more energy from grid than maximum
%load
if (Powergrid>(maxL))
cost=cost+PLCost*(Powergrid-maxL);  
end

%penalize paying more for grid energy than maximum

%do not allow discharging if energy from wind is
%enough
%to meet load
if (((L-Power)<=0)&&(x<0))
cost=Inf;
end
%value from next period
NextVal=0;

%add value from future time periods
for w=0:1:Vmax
    %index for next period Wind
evnext=w+1;

    %add value from future state weighted by
    %probability of given wind speed
    NextVal=NextVal+Windprob(vcurr,vnext)*Value(t+1,roundrnext,vnext);
end

%find value of given state and decision
val=cost+NextVal;

%find value if no discharging allowed
if (x<0)
    valxnd=Inf;
else
    valxnd=val;
end

%if value is less than current minimum value, update
%minimum value, minimum cost, and minimum decision
if (val<minval)
    minval=val;
    mincost=cost;
    minx=x;
end

%if value with no discharging is less than current
%minimum value with no discharging, update minimum
%value and minimum decision
if (valxnd<minvalxnd)
    minvalxnd=valxnd;
    minxnd=x;
end

end

%set value of state to minimum value
Value(t,rcurr,vcurr)=minval;

%set decision of state to minimum decision
\[ X(t, r_{curr}, v_{curr}) = \min x; \]

% set decision with no discharging to minimum
\[ XND(t, r_{curr}, v_{curr}) = \min x_{nd}; \]

% set cost of state to minimum cost
\[ \text{Cost}(t, r_{curr}, v_{curr}) = \min \text{cost}; \]

end
end

%--------------------------------------------------------------------
%
%Inputs for simulation
%initial wind speed (in m/s)
\[ V_i = 5; \]

%number of sample paths
\[ M = 1000; \]

%Setup for simulation
%matrices for statistics of each sample path
%total energy purchased from grid (in MWh)
\[ \text{SimTEG} = \text{zeros}(M, 1); \]
\[ \text{SimTEGD} = \text{zeros}(M, 1); \]
\[ \text{SimTEGC} = \text{zeros}(M, 1); \]
\[ \text{SimTEGN} = \text{zeros}(M, 1); \]

%total cost of energy purchased from grid (in $)
\[ \text{SimTCG} = \text{zeros}(M, 1); \]
\[ \text{SimTCGD} = \text{zeros}(M, 1); \]
\[ \text{SimTCGC} = \text{zeros}(M, 1); \]
\[ \text{SimTCGN} = \text{zeros}(M, 1); \]

%80 percentile of energy purchased from grid (in MWh)
\[ \text{Sim80EG} = \text{zeros}(M, 1); \]
\[ \text{Sim80EGD} = \text{zeros}(M, 1); \]
\[ \text{Sim80EGC} = \text{zeros}(M, 1); \]
Sim80EGN=zeros(M,1);

% 80 percentile of cost of energy purchased from grid (in MWh)
Sim80CG=zeros(M,1);
Sim80CGD=zeros(M,1);
Sim80CGC=zeros(M,1);
Sim80CGN=zeros(M,1);

% 90 percentile of energy purchased from grid (in MWh)
Sim90EG=zeros(M,1);
Sim90EGD=zeros(M,1);
Sim90EGC=zeros(M,1);
Sim90EGN=zeros(M,1);

Sim90CG=zeros(M,1);
Sim90CGD=zeros(M,1);
Sim90CGC=zeros(M,1);
Sim90CGN=zeros(M,1);

% 95 percentile of energy purchased from grid (in MWh)
Sim95EG=zeros(M,1);
Sim95EGD=zeros(M,1);
Sim95EGC=zeros(M,1);
Sim95EGN=zeros(M,1);

Sim95CG=zeros(M,1);
Sim95CGD=zeros(M,1);
Sim95CGC=zeros(M,1);
Sim95CGN=zeros(M,1);

% worst case energy purchased from grid (in MWh)
SimPEG=zeros(M,1);
SimPEGD=zeros(M,1);
SimPEGC=zeros(M,1);
SimPEGN=zeros(M,1);

% worst case cost of energy purchased from grid (in $)
SimPCG=zeros(M,1);
SimPCGD=zeros(M,1);
SimPCGC=zeros(M,1);
SimPCGN=zeros(M,1);
SimPCGN=zeros(M,1);
% total percent below goal level (in percent)
SimTPBG=zeros(M,1);
SimTPBGD=zeros(M,1);

% find energy from average wind speed of 5 m/s (in MWh)
for t=1:T
    SimEWA(t)=Pturb(Avwind+1)*NumTurb;
end

% Simulate the policy over M sample paths
for m=1:M

    % find initial state, decision, and costs
    SimBat(1)=Bi;
    SimBatD(1)=Bi;
    SimSmC(1)=0;
    SimWS(1)=Vi;
    SimEW(1)=Pturb(SimWS(1)+1)*NumTurb;
    SimCW(1)=Pwind*SimEW(1);
    SimPBG(1)=max(G(1)*Bscale-SimBat(1),0)/Bscale*100;
    SimPBGD(1)=max(G(1)*Bscale-SimBatD(1),0)/Bscale*100;
    SimCP(1)=Penalty(1,Bi+1);
    SimX(1)=X(1,Bi+1,Vi+1);
    SimEX(1)=-(SimX(1)/Bscale)*BatCap;
    SimEWX(1)=SimEW(1)+SimEX(1);
    SimED(1)=-(Drivingimpact(1)/Bscale)*BatCap;
    SimXND(1)=XND(1,Bi+1,Vi+1);
    SimL(1)=Load(1);
    SimLX(1)=SimL(1)-SimEX(1);
    SimEG(1)=max(SimL(1)-SimEWX(1)-SimEX(1),0);
    SimCG(1)=Pricegrid(1)*SimEG(1);
    SimEGN(1)=max(SimL(1)-SimEW(1),0);
    SimCGN(1)=Pricegrid(1)*SimEGN(1);
    SimEGC(1)=SimEGN(1);
    SimCGC(1)=SimCGN(1);
    SimEGD(1)=max(SimL(1)-SimEW(1)+SimXND(1)/Bscale*BatCap,0);
    SimCGD(1)=Pricegrid(1)*SimEGD(1);

SimCost(1)=Cost(1,Bi+1,Vi+1);
SimTC(1)=SimCost(1);

for t=2:T

%hour corresponding to time t-1
ht1=rem(t-2,H)+1;

%hour corresponding to time t
h=rem(t-1,H)+1;

%choose a random number
rnum=rand(1);

%upper cdf bound used to correspond random number with a particular choice
ubcdf=0;

%use probability matrix to simulate wind speed this time period
%(in m/s)
for i=0:1:Vmax
    ubcdf=ubcdf+Windprob(SimWS(t-1)+1,i+1);
    if (rnum<=ubcdf)
        SimWS(t)=i;
        break
    end
end

%update battery level (in tenth of percent)
if (SimX(t-1)>0)
    SimBat(t)=SimBat(t-1)+ChE*SimX(t-1)-Drivingimpact(ht1);
else
    SimBat(t)=SimBat(t-1)+SimX(t-1)/DisE-Drivingimpact(ht1);
end

%round the simulated battery level
roundsimbat=round(SimBat(t));

%find test decision for this state (in tenth of percent)
testX=X(t,roundsimbat+1,SimWS(t)+1);

%change rounded simulated battery level if test decision causes battery level to go out of bounds
if (testX>0)
    testbat=SimBat(t)+ChE*testX-Drivingimpact(h);
else
testbat=SimBat(t)+testX/DisE-Drivingimpact(h);
end

if (testbat>Bscale)
  roundsimbat=roundsimbat+1;
elseif (testbat<Bmin)
  roundsimbat=roundsimbat-1;
end

% find decision for this state (in tenth of percent)
SimX(t)=X(t,roundsimbat+1,SimWS(t)+1);

% update battery level of case without discharging (in tenth of percent)
if (SimXND(t-1)>0)
    SimBatD(t)=SimBatD(t-1)+ChE*SimXND(t-1)-Drivingimpact(ht1);
else
    SimBatD(t)=SimBatD(t-1)+SimXND(t-1)/DisE-Drivingimpact(ht1);
end

% round the simulated battery level without discharging
roundsimbatxnd=round(SimBatD(t));

% find test decision for this state (in tenth of percent)
testXND=XND(t,roundsimbatxnd+1,SimWS(t)+1);

% change rounded simulated battery level if test decision causes battery level to go out of bounds
if (testXND>0)
    testbatxnd=SimBatD(t)+ChE*testXND-Drivingimpact(h);
else
    testbatxnd=SimBatD(t)+testXND/DisE-Drivingimpact(h);
end

if (testbatxnd>Bscale)
    roundsimbatxnd=roundsimbatxnd+1;
elseif (testbatxnd<Bmin)
    roundsimbatxnd=roundsimbatxnd-1;
end

% find decision for this state (in tenth of percent)
SimXND(t)=XND(t,roundsimbatxnd+1,SimWS(t)+1);

% update total level charged or discharged (in tenth of percent)
SimSmC(t)=SimBat(t)-Bi;
%find energy from wind this hour (in MWh)
SimEW(t)=Pturb(SimWS(t)+1)*NumTurb;

%find costs of purchasing energy from wind this hour (in $)
SimCW(t)=Pwind*SimEW(t);

%find percent below goal level each time period (in percent)
SimPBG(t)=max(G(h)*Bscale-SimBat(t), 0)/Bscale*100;

%find percent below goal level if no discharging allowed (in percent)
SimPBGD(t)=max(G(h)*Bscale-SimBatD(t), 0)/Bscale*100;

%find costs from penalty this hour (in $)
SimCP(t)=Penalty(t, roundsimbat+1);

%update costs (in $)
SimCost(t)=Cost(t, roundsimbat+1, SimWS(t)+1);
SimTC(t)=SimTC(t-1)+SimCost(t);

%find energy from decision (in MWh)
SimEX(t)=-(SimX(t)/Bscale)*BatCap;

%find energy from grid and cars (in MWh)
SimEWX(t)=SimEW(t)+SimEX(t);

%find energy drained from driving (in MWh)
SimED(t)=-(Drivingimpact(h)/Bscale)*BatCap;

%find non-vehicular load (in MW)
SimL(t)=Load(h);

%find total load (in MW)
SimLX(t)=SimL(t)-SimEX(t);

%find energy from grid this hour (in MWh)
SimEG(t)=max(SimL(t)-SimEW(t)-SimEX(t), 0);

%find costs of purchasing energy from grid this hour (in $)
SimCG(t)=Pricegrid(h)*SimEG(t);

%find energy from grid if there were no cars (in MWh)
SimEGN(t)=max(SimL(t)-SimEW(t), 0);

%find cost of energy from grid if there were no cars (in $)
SimCGN(t)=Pricegrid(h)*SimEGN(t);

%find energy from grid if cars are charged immediately after
%being driven (in MWh)
SimEGC(t) = \max(SimL(t) - SimEW(t) + \text{(Drivingimpact}(ht_1)/Bscale)*\text{BatCap}, 0);

% find cost of energy fromm grid if cars are charged immediately
% after being driven (in $)
SimCGC(t) = \text{Pricegrid}(h)*\text{SimEGC}(t);

% find energy from grid if no discharging allowed (in MWh)
SimEGD(t) = \max(SimL(t) - SimEW(t) + SimXND(t)/Bscale*\text{BatCap}, 0);

% find cost of energy fromm grid if no discharging allowed (in $)
SimCGD(t) = \text{Pricegrid}(h)*\text{SimEGD}(t);

end

% hour corresponding to time T
hT = \text{rem}(T-1, H)+1;

% update battery level (in tenth of percent)
if (SimX(T) > 0)
    SimBat(T+1) = SimBat(T) + \text{ChE}\cdot\text{SimX}(T) - \text{Drivingimpact}(hT);
else
    SimBat(T+1) = SimBat(T) + \text{SimX}(T)/\text{DisE} - \text{Drivingimpact}(hT);
end

% update total level charged or discharged (in tenth of percent)
SimSmC(T+1) = SimBat(T+1) - Bi;

% total energy purchased from grid (in MWh)
SimTEG(m) = \text{sum}(SimEG);
SimTEGD(m) = \text{sum}(SimEGD);
SimTEGC(m) = \text{sum}(SimEGC);
SimTEGN(m) = \text{sum}(SimEGN);

% total cost of energy purchased from grid (in $)
SimTCG(m) = \text{sum}(SimCG);
SimTCGD(m) = \text{sum}(SimCGD);
SimTCGC(m) = \text{sum}(SimCGC);
SimTCGN(m) = \text{sum}(SimCGN);

% 80 percentile of energy purchased from grid (in MWh)
Sim80EG(m) = \text{prctile}(SimEG, 80);
Sim80EGD(m) = \text{prctile}(SimEGD, 80);
Sim80EGC(m) = \text{prctile}(SimEGC, 80);
Sim80EGN(m) = \text{prctile}(SimEGN, 80);

% 80 percentile of cost of energy purchased from grid (in MWh)
Sim80CG(m)=prctile(SimCG, 80);
Sim80CGD(m)=prctile(SimCGD, 80);
Sim80CGC(m)=prctile(SimCGC, 80);
Sim80CGN(m)=prctile(SimCGN, 80);

%90 percentile of energy purchased from grid (in MWh)
Sim90EG(m)=prctile(SimEG, 90);
Sim90EGD(m)=prctile(SimEGD, 90);
Sim90EGC(m)=prctile(SimEGC, 90);
Sim90EGN(m)=prctile(SimEGN, 90);

%90 percentile of cost of energy purchased from grid (in MWh)
Sim90CG(m)=prctile(SimCG, 90);
Sim90CGD(m)=prctile(SimCGD, 90);
Sim90CGC(m)=prctile(SimCGC, 90);
Sim90CGN(m)=prctile(SimCGN, 90);

%95 percentile of energy purchased from grid (in MWh)
Sim95EG(m)=prctile(SimEG, 95);
Sim95EGD(m)=prctile(SimEGD, 95);
Sim95EGC(m)=prctile(SimEGC, 95);
Sim95EGN(m)=prctile(SimEGN, 95);

%95 percentile of cost of energy purchased from grid (in MWh)
Sim95CG(m)=prctile(SimCG, 95);
Sim95CGD(m)=prctile(SimCGD, 95);
Sim95CGC(m)=prctile(SimCGC, 95);
Sim95CGN(m)=prctile(SimCGN, 95);

%worst case energy purchased from grid (in MWh)
SimPEG(m)=max(SimEG);
SimPEGD(m)=max(SimEGD);
SimPEGC(m)=max(SimEGC);
SimPEGN(m)=max(SimEGN);

%worst case cost of energy purchased from grid (in $)
SimPCG(m)=max(SimCG);
SimPCGD(m)=max(SimCGD);
SimPCGC(m)=max(SimCGC);
SimPCGN(m)=max(SimCGN);

%total percent below goal level (in percent)
SimTPBG(m)=sum(SimPBG);
SimTPBGD(m)=sum(SimPBGD);

end
%averages and maxima of statistics

%average total energy purchased from grid (in MWh)
SmaTEG=mean(SimTEG);
SmaTEGD=mean(SimTEGD);
SmaTEGC=mean(SimTEGC);
SmaTEGN=mean(SimTEGN);

%average total cost of energy purchased from grid (in $)
SmaTCG=mean(SimTCG);
SmaTCGD=mean(SimTCGD);
SmaTCGC=mean(SimTCGC);
SmaTCGN=mean(SimTCGN);

%average 80 percentile of energy purchased from grid (in MWh)
Sma80EG=mean(Sim80EG);
Sma80EGD=mean(Sim80EGD);
Sma80EGC=mean(Sim80EGC);
Sma80EGN=mean(Sim80EGN);

%average 80 percentile of cost of energy purchased from grid (in MWh)
Sma80CG=mean(Sim80CG);
Sma80CGD=mean(Sim80CGD);
Sma80CGC=mean(Sim80CGC);
Sma80CGN=mean(Sim80CGN);

%average 90 percentile of energy purchased from grid (in MWh)
Sma90EG=mean(Sim90EG);
Sma90EGD=mean(Sim90EGD);
Sma90EGC=mean(Sim90EGC);
Sma90EGN=mean(Sim90EGN);

%average 90 percentile of cost of energy purchased from grid (in MWh)
Sma90CG=mean(Sim90CG);
Sma90CGD=mean(Sim90CGD);
Sma90CGC=mean(Sim90CGC);
Sma90CGN=mean(Sim90CGN);

%average 95 percentile of energy purchased from grid (in MWh)
Sma95EG=mean(Sim95EG);
Sma95EGD=mean(Sim95EGD);
Sma95EGC=mean(Sim95EGC);
Sma95EGN=mean(Sim95EGN);

%average 95 percentile of cost of energy purchased from grid (in MWh)
Sma95CG=mean(Sim95CG);
Sma95CGD=mean(Sim95CGD);
Sma95CGC=mean(Sim95CGC);
Sma95CGN=mean(Sim95CGN);

%average worst case energy purchased from grid (in MWh)
SmaPEG = mean(SimPEG);
SmaPEGD = mean(SimPEGD);
SmaPEGC = mean(SimPEGC);
SmaPEGN = mean(SimPEGN);

% average worst case cost of energy purchased from grid (in $)
SmaPCG = mean(SimPCG);
SmaPCGD = mean(SimPCGD);
SmaPCGC = mean(SimPCGC);
SmaPCGN = mean(SimPCGN);

% maximum worst case energy purchased from grid (in MWh)
SmmPEG = max(SimPEG);
SmmPEGD = max(SimPEGD);
SmmPEGC = max(SimPEGC);
SmmPEGN = max(SimPEGN);

% maximum worst case cost of energy purchased from grid (in $)
SmmPCG = max(SimPCG);
SmmPCGD = max(SimPCGD);
SmmPCGC = max(SimPCGC);
SmmPCGN = max(SimPCGN);

% average total percent below goal level (in percent)
SmaTPBG = mean(SimTPBG);
SmaTPBGD = mean(SimTPBGD);

% compile statistics
CompStat = zeros(13, 4);
CompStat(1, 1) = SmaTEG;
CompStat(1, 2) = SmaTEGD;
CompStat(1, 3) = SmaTEGC;
CompStat(1, 4) = SmaTEGN;

CompStat(2, 1) = SmaTCG;
CompStat(2, 2) = SmaTCGD;
CompStat(2, 3) = SmaTCGC;
CompStat(2, 4) = SmaTCGN;

CompStat(3, 1) = Sma80EG;
CompStat(3, 2) = Sma80EGD;
CompStat(3, 3) = Sma80EGC;
CompStat(3, 4) = Sma80EGN;

CompStat(4, 1) = Sma80CG;
CompStat(4, 2) = Sma80CGD;
CompStat(4, 3) = Sma80CGC;
CompStat(4, 4) = Sma80CGN;

CompStat(5, 1) = Sma90EG;
CompStat(5, 2) = Sma90EGD;
CompStat(5, 3) = Sma90EGC;
CompStat(5, 4) = Sma90EGN;

CompStat(6, 1) = Sma90CG;
CompStat(6, 2) = Sma90CGD;
CompStat(6, 3) = Sma90CGC;
CompStat(6, 4) = Sma90CGN;

CompStat(7, 1) = Sma95EG;
CompStat(7, 2) = Sma95EGD;
CompStat(7, 3) = Sma95EGC;
CompStat(7, 4) = Sma95EGN;

CompStat(8, 1) = Sma95CG;
CompStat(8, 2) = Sma95CGD;
CompStat(8, 3) = Sma95CGC;
CompStat(8, 4) = Sma95CGN;

CompStat(9, 1) = SmaPEG;
CompStat(9, 2) = SmaPEGD;
CompStat(9, 3) = SmaPEGC;
CompStat(9, 4) = SmaPEGN;

CompStat(10, 1) = SmaPCG;
CompStat(10, 2) = SmaPCGD;
CompStat(10, 3) = SmaPCGC;
CompStat(10, 4) = SmaPCGN;

CompStat(11, 1) = SmmPEG;
CompStat(11, 2) = SmmPEGD;
CompStat(11, 3) = SmmPEGC;
CompStat(11, 4) = SmmPEGN;

CompStat(12, 1) = SmmPCG;
CompStat(12, 2) = SmmPCGD;
CompStat(12, 3) = SmmPCGC;
CompStat(12, 4) = SmmPCGN;
CompStat(13, 1) = SmaTPBG;
CompStat(13, 2) = SmaTPBGD;
Sources of Data

Hourly Wind Data:

PJM Hourly Spot Price Data:

Hourly Demand Data:
This data was obtained from CASTLE Lab. The original source was an electric utility.
References


