NATURAL GAS POWER GENERATION IN THE PRESENCE OF WIND:

A MIXED INTEGER LINEAR PROGRAMMING APPROACH TO THE HOUR-AHEAD UNIT COMMITMENT PROBLEM

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DEPARTMENT OF OPERATIONS RESEARCH AND FINANCIAL ENGINEERING

PRINCETON UNIVERSITY
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Abstract

Power generation is complex because available wind and demand for electric power are each stochastic and difficult to forecast accurately. The power output of coal generators is difficult to change in a short time horizon due to their long minimum warm-up times. Wind is too volatile to be a dependable, short-horizon source of power. Regional Transmission Organizations such as PJM Interconnection, therefore, turn to natural gas as an effective source of short-term power. This thesis focuses on hour-ahead optimization of natural gas generators to supplement day-ahead coal and wind generation. A mixed integer linear programming approach is used to solve the hour-ahead unit commitment problem, which gives an adjusted generation schedule in five minute increments. When the amount of wind power in the system is increased from 5.2% to 20.4% to 40.0%, generation costs decrease and shortage penalties generally increase. A heuristic that increases the effective horizon of the model decreases the total cost. This thesis illustrates how PJM can operate its power market more efficiently while increasing its use of wind power.
Acknowledgements

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To Mom and Dad
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Chapter I

1 Introduction to the Hour-Ahead Unit Commitment Problem

In 2010, total domestic energy production in the U.S. was 22.1 trillion kilowatt hours, of which 17.2 trillion kW·h came from fossil fuel sources and 2.3 trillion kW·h came from renewable energy sources (U.S. EIA, 2012). Of fossil fuel production, about 37.75% came from natural gas sources and 37.71% from coal sources. In addition, domestic energy production is projected to grow by a compound annual growth rate of almost 1.09% from 2010 through 2025, reaching 26.0 trillion kW·h. During this 15 year time period, energy production from coal sources is forecasted to increase by 1.9% and energy production from natural gas sources by 20.5%. Energy is crucial to the functioning of the U.S. economy, with total consumption in 2010 equivalent to about 19.3% of real GDP measured in 2010 dollars (U.S. EIA, 2012; Bureau, 2010).

Energy consumption in the U.S. is dependent on regional transmission organizations (RTOs), which are power grid operators that are responsible for coordinating the generation and sale of electric power across interstate borders. RTOs create a daily generation schedule that determines how much power each generator in the system must produce to ensure that consumers have enough to use on a daily basis. One such operator is PJM Interconnection LLC (PJM), which was founded in 1927 and became the nation’s first fully functioning RTO in 2001.
An independent and neutral party, PJM oversees and operates a competitive wholesale power market that consists of about 1000 nuclear, coal, hydro, and gas generators. PJM also maintains the higher-voltage transmission grid that covers 13 Mid-Atlantic states and the District of Columbia, which provides electricity to over 58 million people (PJM, 2012). Figure 1-1 illustrates the extent of PJM’s service territory. This thesis focuses on how PJM creates and adjusts its daily generation schedule to satisfy consumer demand while minimizing total cost.

Figure 1-1: Diagram of PJM's Service Territory (PJM, 2012)

1.1 PJM’s Two-Phase Problem

PJM’s operation of the electric power market consists of two phases. This section describes each phase and explains why PJM’s approach to solving the problem may have to change due to the projected increase in wind power usage.

1.1.1 The Day-Ahead Problem

In the first phase of simultaneous optimal auction, each generator operator in the PJM network submits a bid indicating the minimum price at which it is willing to generate power for the following day. Operators submit different bids based on parameters inherent to each generator. These parameters include, but are not limited to,
the maximum and minimum output capacities, the variable cost for supplying power (which depends on the generator’s fuel type), and the ramp rate (i.e. the maximum hourly increase in MW production). PJM takes these factors into consideration and creates a schedule for the following day that indicates which generators will turn on each hour and how much power they will produce. PJM makes this schedule by solving the unit commitment problem, which can be formulated as a mixed integer problem where the objective is often to minimize total system costs of generation (Yan and Stern, 2002). This problem is also known as the day-ahead problem.

The day-ahead problem is stochastic due to the uncertain nature of demand. PJM creates the generation schedule to satisfy predicted consumer demand for electric power, but due to weather or other random fluctuations, actual demand in the following day may be different from forecasted demand. Figure 1-2 shows the difference between forecasted and actual demand, where the dotted purple line represents the day-ahead forecast, the dotted turquoise line represents the hour-ahead forecast, and the solid blue line represents actual demand.

![Figure 1-2: Forecasted vs. Actual Demand (California ISO, 2012)](image-url)
1.1.2 The Hour-Ahead Problem

Due to the difference between actual and predicted demand, PJM must adjust its day-ahead schedule in real-time to avoid shortages and displeased consumers. The process of real-time adjustment is the second phase of PJM’s problem. It is also known as the hour-ahead problem. PJM makes adjustment decisions every five minutes, turning on or off the fast generators to make up the difference between actual and forecasted demand (Botterud et al., 2010). To minimize total system costs, PJM satisfies demand by using the cheapest generators that are able to operate within these short time horizons. When demand forecasts are relatively accurate, PJM avoids brownouts and unhappy consumers. The stochastic nature of the hour-ahead problem, however, increases significantly when PJM allocates larger portions of the generation schedule to wind power. Accurate wind forecasts are harder to obtain than demand forecasts as wind is more volatile. But since the U.S. Energy and Information Administration forecasts wind generation capacity to grow at an annual rate of 2.2% between 2010 and 2025, RTOs such as PJM must improve their methods of solving the day-ahead and hour-ahead problems (2012).

1.2 The Impact of Wind Power: 20% Wind by 2030

In 2008, the U.S. Department of Energy published a report studying whether wind could feasibly account for 20% of the total U.S. power supply by 2030. This percentage of wind integration into the power supply is called wind penetration. Wind accounted for 11.5% of renewable energy production in 2010 but only 1.2% of total energy production (U.S. EIA, 2012). The use of renewable energy for power generation, however, is on the
rise. The U.S. Energy Information Administration projects total renewable energy
generation to grow at an annual rate of 1.1% from 2010 to 2025 to 148.42 GW (2012).
Figure 1-3 shows the breakdown by fuel type of North American generators that are
under construction in 2011; the x-axis indicates the year in which a generator will first go
online and the y-axis represents total capacity.

![Figure 1-3: Future Generator Breakdown by Fuel Type (Berst, 2011)](image)

1.2.1 Potential Benefits

The 2008 Department of Energy report assumes that U.S. electricity consumption
will increase 39% from 2005 to 2030. It also assumes that by 2030 wind turbine energy
production will increase by 15% and turbine costs will decrease by 10%, while costs and
performance levels of fossil fuel technologies stay constant. Achieving the 20% benchmark
would require U.S. wind generation capacity to increase from 11.6 gigawatts (GW) in 2006
to 305 GW in 2030 (U.S. DOE, 2008). In contrast, total wind generation capacity currently
is projected to reach only 57 GW by 2030 (U.S. EIA, 2012).
Reaching the 20% benchmark does not incur significant marginal costs. Even if wind generation capacity is not increased, additional infrastructure is nonetheless needed to satisfy the growth in electricity consumption by 2030. The marginal cost of increasing wind capacity is $43 billion, or approximately $0.50 per household per month (U.S. DOE, 2008).

On the other hand, increasing wind penetration to 20% by 2030 would reduce carbon emissions by 825 million tons per year, which would save between $50 and $145 billion in regulatory costs. The plan would lead to an eight percent reduction in water consumption – cumulatively saving four trillion gallons of water – as well as an 11% reduction in nationwide use of natural gas power and an 18% reduction in coal power.

Figure 1-4 shows how 46 states will have established substantial wind presence by 2030 under the plan and how eight states will each have wind capacity greater than ten GW. The concentration of offshore wind farms (denoted by the blue icons) along the Mid-Atlantic is consistent with Google’s recent investment in the Atlantic Wind Connection, which is a proposed transmission backbone along the Mid-Atlantic that will connect future offshore wind farms (Wald, 2010).
Wind is a promising source of renewable energy. Regardless of whether the plan is implemented, wind will play an increasingly large role in U.S. power generation. PJM and other RTOs, therefore, must improve their unit commitment models, which are essential to the efficient operation of an electric power market.

1.2.2 Implications for RTOs

The integration of more wind power into the power grid would require operational changes because “other units in the power system have to be operated more flexibly to maintain the stability of the power system” (Barth et al., 2006). With offshore wind, for example, the expansion of transmission grids in remote regions would be necessary to avoid bottlenecks in wind power delivery. In addition, the system may require more spinning reserves – the ability of online backup generators to produce power at an instant’s notice – in case the wind suddenly disappears. These uncertainties would lead to shifts in supply and demand and affect market clearing prices (Barth et al., 2006).

Wind volatility is not just a problem in the literature; it already has significant real-life implications. Texas, for example, currently has about 10 GW of wind generation capacity, but it sometimes provides only 0.88 GW of power (Bryce, 2011). The large volatility of wind increases the difficulty of the day-ahead and hour-ahead problems. RTOs may need to create generation schedules in which the total output varies between extreme values in short amounts of time in order to mimic wind fluctuations.

The bigger problem, however, is the unpredictability of wind. If wind were volatile but deterministic, solving the unit commitment model would create an effective schedule given enough generators. But because current wind forecasts are inaccurate,
solving the day-ahead problem is not enough. To understand why, it is useful to conduct a literature review of the unit commitment problem.

1.3 Review of the Unit Commitment Problem

Padhy defines unit commitment as the “problem of determining the schedule of generating units within a power system, subject to device and operating constraints” (2004). Methods of solving the problem range from simplistic approaches such as brute force, in which all possible solutions are listed and the best is chosen, to new algorithms such as shuffled frog leaping, an evolutionary algorithm with an especially high convergence speed (Ebrahimi et al., 2011). Although the unit commitment problem experiences ongoing research activity, all algorithms for the problem involve optimizing an objective function subjective to multiple constraints.

1.3.1 Common Objective Functions and Constraints

In the literature, the objective of the unit commitment model is usually to minimize total system costs of generation (Padhy, 2004):

\[
\min \sum_{i=1}^{N} \sum_{t=1}^{T} C_{i,t}(p_{i,t}) + K_{i,t}
\]

Here, \( p_{i,t} \) is the output of generator \( i \) at time \( t \), \( C_{i,t}(p_{i,t}) \) is the cost of generator \( i \) outputting \( p_{i,t} \), and \( K_{i,t} \) is the fixed cost of generator \( i \) at time \( t \). The system has \( N \) generators, and the problem is solved for \( T \) time periods. The costs associated with \( C_{i,t}(\cdot) \) include the fuel cost, which is normally modeled as a quadratic, and the maintenance cost, which is usually linear. Fixed cost \( K_{i,t} \) typically includes start-up costs.
and shut-down costs. The constraints typically involve the generators’ minimum online times, minimum off times, and maximum ramp rates (Padhy, 2004).

Alternatively, in the case of deregulated electricity markets, the objective function can be to maximize profit (Padhy, 2004):

\[
\text{max} \sum_{i=1}^{N} \sum_{t=1}^{T} p_{i,t} \times \bar{P}_{i,t} \times I_{(i,t)} - (C_{i,t}(p_{i,t}) + K_{i,t})
\]

Here, \(\bar{P}_{i,t}\) is the time zero forecasted price of generator \(i\)'s incremental output at time \(t\) and \(I_{(i,t)}\) is an indicator variable that is equal to 1 when generator \(i\) is online at time \(t\) and 0 otherwise. The expression \(p_{i,t} \times \bar{P}_{i,t} \times I_{(i,t)}\) refers to the revenue earned by generator \(i\) at time \(t\), and the expression in parenthesis is the operating cost from the prior formulation (Padhy, 2004). The constraints are the same as before. In either case, the problem can be augmented with grid constraints that restrict the amount of output flowing from a generator in one location to the demand in another location. The implementation of grid constraints requires additional information about the distances and maximum flow capacities between generator and demand locations.

Yan and Stern propose an objective function that uses the marginal clearing price \(c_{i,t}^m\), which is independent of generator \(i\) (2002):

\[
\text{min} \sum_{i=1}^{N} \sum_{t=1}^{T} c_{i,t}^m p_{i,t} + K_{i,t}
\]

Here, \(K_{i,t}\) is the startup cost of generator \(i\) at time \(t\), and \(c_{i,t}^m\) is the maximum time \(t\) fuel cost for generator \(i\) over the set of all online generators at time \(t\). This objective function analyzes the problem from the perspective of the market clearing price instead.
of the traditional bid price. A limitation of this functional form is that it loses the separable structure required by the common Lagrangian relaxation algorithm.

1.3.2 Classes of Algorithm

The unit commitment problem is a mixed integer, nonlinear problem with many approximate solutions (Chang et al., 2004). Padhy classified common algorithms for the problem into 16 types (2004):

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<td>1. Exhaustive Enumeration</td>
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<td>3. Dynamic Programming</td>
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<td>4. Linear Programming</td>
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<td>5. Branch and Bound</td>
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<td>6. Lagrangian Relaxation</td>
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<td>7. Interior Point Optimization</td>
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<td>15. Ant Colony Search</td>
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<tr>
<td>16. Hybrid Models</td>
</tr>
</tbody>
</table>

In practice, the most commonly used classes are priority listing, linear programming, and Lagrangian relaxation, perhaps due to their limited complexity and ease of implementation. Priority listing creates a list of all generators sorted from least expensive fuel cost to most expensive fuel cost; the algorithm ramps up the output of the cheapest generators until demand is satisfied. Linear programming approximates the objective function and constraints as linear functions and constraints, which reduces the problem to a linear optimization problem. Lagrangian relaxation rewrites the constraints using Lagrange multipliers to add penalty terms, and the algorithm relaxes successive
constraints to reach an optimal solution. Lagrangian relaxation traditionally has been the most common method because it is easy to customize individual constraints for generators with unique characteristics, but it is not the method used in this thesis (Padhy, 2004).

1.3.3 Justification of Mixed Integer Linear Programming

The issue of size typically has hindered the use of linear programming to solve the unit commitment problem, since in the worst case scenario the running time is $O(N^3)$. The development of more efficient optimization packages, however, has refueled interest in both linear programming and mixed integer linear programming, which is linear programming using a mixture of integer and non-integer variables. Advantages of using mixed integer linear programming include the relatively noncomplex process of linearizing the constraints and the use of dual variables to give additional information on pricing (Chang et al., 2004).

This thesis, therefore, formulates the hour-ahead unit commitment problem as a mixed integer linear program. Code written in JAVA creates the linear program and calls the optimization package CPLEX to solve it. The notation for this thesis’s model comes from Chang et al.’s formulation, which minimizes system costs while writing constraints as linear equations with integer variables (2004).

Consider as an example the constraint that prevents generator $i$ from turning on and off at the same instant. Let $u_{i,t}$ be an indicator variable that is 1 when generator $i$ is online at time $t$ and 0 otherwise. Takriti et al.’s traditional Lagrangian relaxation model implements this constraint by adding a penalty term to the objective function (1996):

$$
\lambda_{i,t} (u_{i,t} - c_{i,t})
$$
Here, $\lambda_{i,t}$ is the Lagrangian multiplier, and $c_{i,t}$ is a probability-weighted average of generator output decisions (Takriti et al., 1996). But the objective function may be quadratic. In contrast, Chang et al. propose the following linear constraints (2004):

$$u_{i,t} - u_{i,t-1} = y_{i,t} - z_{i,t}$$

$$y_{i,t} + z_{i,t} \leq 1$$

Here, $y_{i,t}$ and $z_{i,t}$ are integer variables corresponding to whether generator $i$ turns on or off at the instant $t$, respectively. These constraints ensure that generator $i$ cannot turn on and off at the same time. They are not added to the original objective function, which stays linear. Jessica Zhou’s senior thesis, which solves PJM’s day-ahead problem and serves as a starting point for this thesis, also uses Chang et al.’s formulation of linear constraints (2010; 2004). Zhou, however, uses the priority listing algorithm to solve the hour-ahead problem (2010). This thesis seeks to make a contribution to the literature by solving the hour-ahead problem in the presence of wind power through mixed integer linear programming.

### 1.4 Overview of Thesis

This chapter introduces PJM’s two-phase problem of simultaneous optimal auction (the day-ahead problem) and real-time adjustment (the hour-ahead problem). It also motivates the need to integrate more wind power into PJM’s system and provides a brief review of the unit commitment problem. Chang et al.’s formulation and Zhou’s model are used as a starting point, although modifications are needed (2004; 2010).

Zhou uses mixed integer linear programming to solve the day-ahead problem and a priority listing algorithm to solve the hour-ahead problem (2010). The latter algorithm
sorts each type of generator from least to most expensive fuel cost. Starting with the least expensive coal generators and ending with the most expensive gas generators, the algorithm increases the outputs of successively more expensive generators of each type until actual demand is cleared (Zhou, 2010).

This algorithm, however, fails to take advantage of the cycling speed of natural gas generators, which can be turned on and off within minutes. Chapter 2 provides an overview of natural gas generators (both combustion turbine and combined cycle power plants) and demonstrates their ability to operate on a smaller time scale, a nuance that is lacking in Zhou’s model:

“The model assumes that at the beginning of each hour, there is a demand deviation, and these demand deviations last for the whole hour. Generators are given only five minutes to ramp up or down to adjust to exogenous demand levels. The generation for the next 55 minutes remains constant until the beginning of the next hour, when new exogenous demand requires portfolio rebalancing again within five minutes.” (Zhou, 2010)

This thesis, therefore, attempts to add to the literature by proposing an hour-ahead model in Chapter 3 that solves PJM’s hour-ahead problem through mixed integer linear programming. For each hour of simulation, the hour-ahead unit commitment problem is solved in 12 five minute increments, which reduces the time scale of the problem to take advantage of the speed of natural gas generators and, therefore, makes the modeling of those generators more realistic.

Figure 1-5: PJM’s Hunterstown Combined Cycle Power Plant in PA (GenOn, 2010)
This thesis also attempts to add to the literature of the unit commitment problem. The formulations of Chang et al. and Zhou do not include a generator warm-up state (2004; 2010). In their models, generators produce zero power only in the off state. In the real world, however, generators must warm up before they go online and produce their first MW of power. The hour-ahead model proposed in Chapter 3 requires each generator to be in exactly one of three states: warming up, online, and off. As a result, this formulation allows the use of a minimum warm-up time in addition to minimum online and off times, which provides a more realistic model for natural gas generators. Chapter 3 also describes the linear constraints required to implement the hour-ahead model as a mixed integer linear program. A model to include combined cycle generators in the hour-ahead model is also presented at the end of Chapter 3, intended as a reference for future research when sufficient data is available.

Chapter 4 presents the mathematical formulation of the simulation model, which relates the hour-ahead model to its day-ahead counterpart. The chapter explains how the hour-ahead model fits into the simulation as a whole.

The three sources of data required for the simulation are reviewed in Chapter 5, which describes where the generator, demand, and wind data are obtained from and how they are adjusted from the original data for use in the simulation. The chapter also describes how the wind penetration simulation parameter is calculated.

Chapter 6 analyzes the results of the simulations. Comparisons of shortage and overage statistics, cost distribution, and generator activity are compared at wind penetration levels of 5.2%, 20.4%, and 40.0%. Increased wind volatility through the use of Brownian bridge simulation is also studied at 5.3% wind.
Chapter 7 proposes a heuristic to amend a limitation of the hour-ahead model, which is explained initially in Chapter 6. The heuristic increases the effective horizon of the model. It is tested at wind penetration levels of 5.2%, 20.4%, and 39.9%, and the results are compared to the base case simulations. The heuristic is also generalized by using a tunable parameter, which is tested at 39.9% wind. A model to increase the horizon by five minutes without using a heuristic is presented at the end of the chapter, intended as a reference for future research.

Finally, Chapter 8 summarizes the conclusions of the simulations, describes limitations of the model, and proposes areas for future research.
Chapter II

2 Details of Natural Gas Generators

Power systems for production of electric power fall into one of four major categories: fossil fuel power plants (which include coal generators and natural gas generators), nuclear power plants, hydraulic power plants, and renewable energy power plants (Boyce, 2010). Natural gas generators can be further separated into two types: combustion turbine (also called gas turbine or simple cycle) generators and combined cycle generators.

Different types of generators are used to satisfy different kinds of demand for power. During the day-ahead bidding process, generators submit a price below which they are unwilling to generate power. PJM aggregates these bids to form the electric power supply curve, known as the merit order. Generators with low marginal costs enter at the bottom of the merit order because they can profit even when the price charged to consumers is low. More expensive generators enter the merit order at higher prices.

Differences in generators’ marginal cost are largely due to fuel type. Natural gas generators incur significantly larger variable costs than their coal generator counterparts (about $0.05/kWh compared to $0.01/kWh) and therefore appear higher in the merit order because they demand a higher price to operate (Rebenitsch, 2011).
2.1 Slow versus Fast Generation

Coal generators are often coal-fueled steam power plants that boil water and use the resulting steam to generate power. These are categorized as slow generators because they tend to have long warm-up periods due to the time it takes to boil the water. Slow generators and others at the bottom of the merit order generally serve baseload—the portion of demand that never falls below a certain baseline even in the early morning or late evening of the day. Baseload generally comprises 30% to 40% of the maximum load for a given time period.

Due to their quick startup times and ramp rates, on the other hand, natural gas generators are categorized as fast generators. They typically are used to satisfy peakload, the portion of demand that fluctuates highly depending on the time of day. They are operated in cycling mode because they can complete multiple cycles of turning on and off in a day and begin generating power on the order of minutes instead of hours.

Although they are extremely costly to operate from a marginal perspective, natural gas generators are less expensive to build (Cordaro, 2008). As shown in Figure 2-1, they have become more common as sources of power consumption in the last 20 years due to their lower carbon emissions and their competitive pricing. The y-axis is in thousands of cubic feet.
From 1998 to 2008, real natural gas prices measured in 2005 dollars increased 204% (U.S. EIA, 2012). Many natural gas generators shut down because they could not compete with older and cheaper coal plants, whose higher rates of carbon emissions were allowed under grandfather clauses (Boyce, 2010). From 2008 to 2010, however, natural gas prices have plummeted 44% (U.S. EIA, 2012). Along with new environmental laws that reduce carbon emissions in power plants, cheaper prices have shifted the national emphasis to natural gas generators (Berst, 2011). About 84% of new U.S. power generation is expected to come from natural gas sources (Boyce, 2010). In fact, at least four M&A, direct equity, and private equity deals over $1 billion were announced in the natural gas space in February and March of 2012 alone (de la Merced, 2012a; de la Merced, 2012b; Roose, 2012; Austen, 2012).

The recent push toward cheap natural gas has led some industry experts to question whether it will hurt the effort to expand the smart grid—the electricity grid that integrates renewable energy and electric vehicles (Berst, 2011). This fear seems
unfounded. Natural gas improves the smart grid by replacing dirtier coal generators, and their correct usage can help PJM integrate more wind into the system. The following sections explore the operational details of both types of natural gas generators.

### 2.2 Combustion Turbine Generators

Combustion turbine generators contain at least one combustion turbine that is used to burn gas and produce energy. Unlike coal-fueled steam generators, no water is necessary to turn the turbine, which is instead turned by the force of burning gas. Figure 2-2 depicts a typical combustion turbine.

![Combustion Turbine Diagram](image)

*Figure 2-2: Components of Combustion Turbine (Fossil, 2011)*

The combustion turbine is similar to a jet engine because it draws air into the engine through the inlet section, which is then pressurized and injected into the combustion chamber at high speeds on the order of hundreds of miles per hour. The air then mixes with the natural gas fuel that is injected into the combustion system. This high-pressure combination burns at about 2300 °F and flows into the turbine, where the resulting force rotates the turbine’s airfoil blades. In addition to generating power, the rotating blades allow waste heat to exit through the exhaust. The higher the temperature
of the combustion turbine generator, the more efficient it is. Some of the critical metal components of the combustion turbine, however, may only withstand temperatures up to 1700 °F before failing. Some injected air, therefore, is diverted to cool the metal, which decreases the plant’s efficiency but prolongs its lifespan (Fossil, 2011).

Since it does not require boiling water, combustion turbine generators can go online and generate power from a cold start in minutes rather than hours. Furthermore, since there is usually no minimum online time for combustion turbine plants – which applies to coal plants to protect the equipment – they can be turned off at a moment’s notice. These characteristics make combustion turbine plants suitable to satisfy peakload, which often appears and disappears within minutes (Tucker et al., 2009). Figure 2-3 depicts a typical combustion turbine generator.

![Figure 2-3: Components of Combustion Turbine Power Plant (Tennessee, 2011)](image)

Note that the waste heat generated in the combustion chambers is simply released from the exhaust. This unused source of energy is the major operational difference between a combustion turbine generator and a combined cycle generator.
2.3 Combined Cycle Generators

Combined cycle generators have at least one gas turbine and at least one steam turbine. Unlike combustion turbine generators, these power plants pass the waste heat from fueling the gas turbine through a heat recovery steam generator to boil water. They use the resulting steam to spin the steam turbine and generate additional power, after which the steam is passed through a condenser and transformed back to water for further steam generation. Combined cycle generators are more efficient than their combustion turbine counterparts because they use the byproduct heat that would otherwise be wasted. The gas turbines typically generate about 60% of the total power while the steam turbines generate 40%, although this ratio depends on the number of turbines (Boyce, 2010). Figure 2-4 illustrates a typical combined cycle generator.

![Components of Combined Cycle Power Plant](image)

Figure 2-4: Components of Combined Cycle Power Plant (Shepard, 2010)

Combined cycle generators can operate as combustion turbine generators only if they install a bypass damper that increases the efficiency of waste heat diversion; doing so without a bypass damper would be inefficient (Kendig, 2011).
2.3.1 Comparison of Efficiency

As shown in Table 2-1, combined cycle generators are more efficient than combustion turbine generators, which in turn are more efficient than coal generators.

<table>
<thead>
<tr>
<th>Generator Type</th>
<th>Variable Costs ($/kW)</th>
<th>Fixed Costs ($/kW)</th>
<th>Heat Rate (Btu/kW·h)</th>
<th>Net Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>3.0</td>
<td>1.43</td>
<td>9749</td>
<td>35%</td>
</tr>
<tr>
<td>Combined Cycle</td>
<td>4.0</td>
<td>0.35</td>
<td>6203</td>
<td>55%</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>5.8</td>
<td>0.23</td>
<td>7582</td>
<td>45%</td>
</tr>
</tbody>
</table>

Combined cycle generators were originally designed to satisfy baseload, but they are better suited to satisfy peakload than coal generators due to their faster ramp rates. They can cycle between 40% and 100% of maximum capacity in a single day and are operated with multiple starts (Boyce, 2010). Compared to combustion turbine generators, however, combined cycle plants are more suited for baseload operation.

2.3.2 The Operation of Combined Cycle Generators

Combined cycle generator operation is illustrated using Gilbert Generating Station in New Jersey, which is part of PJM. This station has a 288 MW, 4x1 combined cycle power plant. The generator’s 288 MW capacity refers to the total generation of its gas and steam turbines. The 4x1 multi-shaft indicator means the combined cycle plant has four gas turbines powering one steam turbine. Using multiple gas turbines to supply steam to each steam turbine generally increases the plant’s efficiency.

Combined cycle plants usually have a separate water boiler for each gas turbine. This design leads to a constant warm-up time for the steam turbine. Regardless of how many gas turbines are initially switched on, in other words, the demineralized water boils
in the same amount of time because each gas turbine has its own separate boiler. The steam turbine warm-up time, therefore, is dependably constant, according to Mike Kendig, the Operations Manager of Hunterstown Generating Station in Gettysburg, PA, allowing plant operators to know exactly how far in advance they must ignite the gas turbines in order to become fully online by a certain time (2011).

The output of the steam turbine, however, will be proportional to the number of online gas turbines. Two online gas turbines will generate half as much steam as four online gas turbines and, therefore, half as much power through the steam turbine. The use of separate boilers is indicated by the $a \times b \times c$ notation, where $a$ is the number of gas turbines, $b$ is the number of boilers, and $c$ is the number of steam turbines (Kendig, 2011). The Gilbert combined cycle plant, therefore, is 4x4x1.

According to Neil MacIntosh, the Plant Manager of Gilbert Generating Station in Milford, NJ, the first step in taking a cold combined cycle power plant to full capacity is to turn on any combination of the gas turbines (2011). Each gas turbine does not immediately produce any power because it must first warm up. During this time, the gas fuel is injected at approximately 45 psi and must be pressurized to approximately 400 psi before the gas turbine can generate power (Borer, 2011). After 15 minutes, each gas turbine goes online and produces its first MW of power. The online gas turbines can then be ramped up to their full capacity according to their ramp rates. For example, each of the four gas turbines in Gilbert Generating Station’s combined cycle power plant has a ramp rate of 2.5 \( \frac{MW}{min} \). The ramping capability of these gas turbines is additive: if exactly three of the gas turbines are online, the generator’s ramp rate would be 7.5 \( \frac{MW}{min} \), and if all four of the gas turbines are online, it would be 10 \( \frac{MW}{min} \) (MacIntosh, 2011).
After the boiler warms up, the waste heat from the gas turbines begins to boil the water. Exiting the gas turbines at almost 1200 °F, the waste heat enters a heat recovery steam generator pictured in Figure 2-5, where it passes over and heats the tubes that contain demineralized water. The majority of this heat is used to boil the water, after which the heat dissipates from the power plant at a considerably lower temperature of around 200 °F (Kehlhofer et al., 2009). The time required to boil the water and generate sufficient steam can be interpreted as a warm-up time for the steam turbine. For the Gilbert Generating Station combined cycle plant, the steam turbine warm-up time is about 3.5 hours (MacIntosh, 2011). This value generally depends on the steaming capacity of the power plant, or the amount of generated steam measured in $\text{lbs/hr}$. The
warm-up time is also limited by the boiler’s rate of temperature increase, usually not exceeding $\frac{100^\circ F}{hr}$ (Borer, 2011).

After the water boils, the steam turbine goes online and begins generating power; it can also be ramped up to its maximum capacity at its ramp rate, which may be different from the gas turbine’s ramp rate. For example, Gilbert Generating Station’s combined cycle plant has a steam turbine ramp rate of $2 \frac{MW}{min}$, 20% lower than that of a gas turbine (MacIntosh, 2011). After the steam turbine is ramped up to its maximum capacity, the entire combined cycle plant operates at full capacity.

At least one gas turbine must be online for the steam turbine to be online; otherwise no steam is available to drive the steam turbine. It is impossible, therefore, to run only the steam turbine component of the combined cycle plant (MacIntosh, 2011).

When operators reduce the power output of an online combined cycle plant, they ramp down any number of the gas turbines, which causes the steam turbine to ramp down simultaneously due to less available generated steam as fuel. Although it is possible for operators to ramp down only the steam turbine – for example, by flipping a switch to reduce the amount of steam introduced to the steam turbine – this is not a common practice (Kendig, 2011). The reason is that zero variable cost is associated with the steam turbine component of the combined cycle plant because its power generation is entirely dependent on the gas turbines and requires no additional cost.

To turn off the combined cycle plant, gas turbines are ramped down to minimum load and subsequently switched off, which causes the steam turbine to be switched off when it is operating at approximately 15% load. Gradually ramping down the steam turbine from 15% load to 0% load is avoided, however, to prevent equipment damage.
The gas turbines generally have a minimum online time in order to preserve the equipment but no minimum off time, allowing the gas turbines to be switched on instantaneously to satisfy peakload. The steam turbine, on the other hand, generally has a minimum off time in order to prevent damage to the steam turbine. It has no minimum online time, however, so given that the gas turbine’s minimum online time is satisfied, operators may shut down the combined cycle plant during the boiling of the demineralized water without damaging the equipment (Kendig, 2011).

2.3.3 Reducing Boiling Time

The limiting factor in how quickly a combined cycle plant can reach full capacity is the boiling time. On the order of six hours for coal generators, the boiling time is reduced in combined cycle generators when applying a technique known as the “steam blanket.” When the gas and steam turbines are off, operators pay a cost to keep the demineralized water at a higher temperature and pressure, which allows it to be boiled
much quicker (MacIntosh, 2011). For example, the combined cycle generator at Gilbert Generating Station has a 3.5 hour boiling time, which is about 1.5 hours to 2 hours shorter than what it would be without the use of a steam blanket (MacIntosh, 2011).

Another technology is the “start-up on the fly” method developed by Siemens. This technique eliminates the steam turbine’s warm-up time by leveraging the cold steam produced by the heat recovery steam generator in a full temperature and pressure environment. A single shaft 400 MW combined cycle plant can go from ignition to full capacity in 40 minutes (Henkel, 2008). Figure 2-7 shows the improvement in combined cycle start-up time.

![Figure 2-7: Benefits of Start-up on the Fly Technology (Henkel, 2008)](image-url)
Chapter III

3 The Hour-Ahead Model

This chapter establishes a mathematical model for the hour-ahead unit commitment problem. It then transcribes that model using linear constraints to solve it as a mixed integer linear program. The modeling of combustion turbine generators is similar to that of coal generators in Jessica Zhou’s senior thesis on the day-ahead problem (2010). The use of warm-up times, however, changes the necessary constraints.

Based on the study of combined cycle generators in the previous chapter, it is possible to model these generators as interconnected gas turbine and steam turbine generators. It is also possible to model them as slow generators because they are generally used to satisfy baseload. The former method requires separate parameter data for each component of the combined cycle plant. The available data, however, does not distinguish between components, so this thesis uses the latter method. The hour-ahead model established in this chapter, therefore, only applies to fast generators. The last section of this chapter proposes a separability model for combined cycle generators as a reference for future research when separate combined cycle data is available.

3.1 Assumptions

Many useful parameters for PJM’s hour-ahead problem are unavailable, and not all of the available parameters are useful in modeling the problem. Assumptions,
therefore, are required to reduce the dimensionality of the hour-ahead model and diminish its runtime to strike a balance between realism and efficiency. These assumptions are listed below.

1. **Use of actual demand:**

   Due to the lack of predicted demand data in five minute increments and the difficulty of simulating greater dependency between sub-hourly predicted and actual demand, actual demand is used as input to the hour-ahead model. Although this implementation allows the model to peek 55 minutes into the future every hour, an eventual switch to using predicted demand is relatively straightforward when the data becomes available.

2. **Limitations of modeling the system:**

   Due to lack of data, certain aspects of PJM’s system are not modeled. These include but are not limited to: supply side offers, transmission and grid constraints, and battery storage.

3. **Limitations of modeling the generators:**

   Due to lack of parameter data, the hour-ahead model does not include certain aspects of generators that may appear in the unit commitment literature. These include but are not limited to: startup and shutdown costs, cold and hot startup conditions, cool-down state, temperature- and season-dependent efficiency, output-dependent ramp rates, and actual generation costs (Padhy 2004; MacIntosh, 2011; Kendig, 2011; Kelhoffer et al., 2009).
4. **No violation of the off–warm-up–online cycle:**

In Zhou’s thesis, generators can go online immediately after satisfying minimum off times (2010). In reality, however, generators must warm up before generating power, even after satisfying minimum off times (Kendig, 2011). At any given time, this model assumes generators are in exactly one of the following three states: warming up, online, and off. If a generator is off, its next state is the warming up state (it cannot directly go online). If a generator is warming up, its next state is the online state (it cannot directly turn off). If a generator is online, its next state is the off state (it cannot directly begin warming up).

5. **No reserve requirement:**

The North American Electric Reliability Corporation (NERC) standards recommend reserving a percentage of maximum demand forecasts (Botterud et al., 2009). The purpose of maintaining reserves is to give generators a buffer during real-time adjustment in case actual demand greatly exceeds predicted demand. Since the hour-ahead model uses actual demands, it does not incorporate reserves.

6. **Combined cycle generators are categorized as slow generators:**

Due to lack of separate data for the gas turbine and steam turbine components of combined cycle generators, combined cycle generators are assumed to be slow generators.
7. **Initial state of generators during the first hour of the first day:**

All generators are assumed to be off prior to the first time period of simulation. They have satisfied their minimum off time, so they may begin warming up during that period but cannot go online. This assumption is guaranteed by the initial hour constraints.

### 3.2 List of Variables

This list contains all variables associated with gas generators that are used in the hour-ahead unit commitment model. Depending on its subscript, each of the following variables and parameters may exist $\forall i \in I^G$, $t' = 1, \ldots, T$ where $I^G$ is the set of all gas generators and $T = 12$ is the number of time increments per hour. In addition, this model uses a lagged information process. The hour-ahead model schedules decisions at a certain time, but these decisions are implemented at later times. The variable $a_{t,t',i}$, therefore, denotes the information or decision regarding generator $i$ that is known or made at time $t$ by the hour-ahead model and is actionable or implemented at time $t' > t$. 
Table 3-1: List of Variables for Hour-Ahead Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decision Variables</strong></td>
<td></td>
</tr>
<tr>
<td>$w_{t,t',i}$</td>
<td>Warming up status: is 1 if combustion turbine generator $i$ is warming up at time $t'$, is 0 otherwise</td>
</tr>
<tr>
<td>$w_{t,t',i}^{on}$</td>
<td>Begin warming up indicator: is 1 if combustion turbine generator $i$ begins warming up at time $t'$, is 0 otherwise</td>
</tr>
<tr>
<td>$u_{t,t',i}$</td>
<td>Online status: is 1 if combustion turbine generator $i$ is online at time $t'$, is 0 otherwise</td>
</tr>
<tr>
<td>$y_{t,t',i}^{on}$</td>
<td>Going online indicator: is 1 if combustion turbine generator $i$ goes online at time $t'$, is 0 otherwise</td>
</tr>
<tr>
<td>$y_{t,t',i}^{off}$</td>
<td>Turning off indicator: is 1 if combustion turbine generator $i$ turns off at time $t'$, is 0 otherwise</td>
</tr>
<tr>
<td>$p_{t,t',i}$</td>
<td>Combustion turbine generator $i$’s committed generation (MW) at time $t'$</td>
</tr>
<tr>
<td>$\epsilon_{t,t'}$</td>
<td>Slack variable representing power shortage at time $t'$</td>
</tr>
<tr>
<td><strong>Generator Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$C_{t,i}^{fuel}$</td>
<td>Variable fuel cost ($\frac{s}{MW}$) for combustion turbine generator $i$ at time $t$</td>
</tr>
<tr>
<td>$p_{i}^{min}$</td>
<td>Minimum output (MW) for combustion turbine generator $i$</td>
</tr>
<tr>
<td>$p_{i}^{max}$</td>
<td>Maximum output (MW) for combustion turbine generator $i$</td>
</tr>
<tr>
<td>$\tau_{i}^{warmup}$</td>
<td>Minimum warm-up time in number of increments for combustion turbine generator $i$; set to be $\geq 1$</td>
</tr>
<tr>
<td>$\tau_{i}^{online}$</td>
<td>Minimum online time in number of increments for combustion turbine generator $i$; set to be $\geq 1$</td>
</tr>
<tr>
<td>$\tau_{i}^{off}$</td>
<td>Minimum off time in number of increments for combustion turbine generator $i$; set to be $\geq 1$</td>
</tr>
<tr>
<td>$\Delta_{i}^{up}$</td>
<td>Ramp-up rate ($\frac{MW}{increment}$) for combustion turbine generator $i$ ($&gt; 0$)</td>
</tr>
</tbody>
</table>

32
\( \Delta_{i}^{\text{down}} \)  
Ramp-down rate \( \frac{\text{MW}}{\text{increment}} \) for combustion turbine generator \( i \) (< 0) 

**System Parameters**  
\( D_{t'} \)  
Actual demand at time \( t' \) that must be satisfied by fast generators (excludes slow generation and wind power) 

\( C_{t'}^{\text{shortage}} \)  
Penalty \( \left( \frac{\$}{\text{MW}} \right) \) for power shortage at time increment \( t' \) 

**Transition Variables**  
\( w_{t,i} \)  
Combustion turbine generator \( i \)'s warming up status at \( t = T' \), the last increment of this hour: is 1 if warming up, 0 if not warming up 

\( u_{t,i} \)  
Combustion turbine generator \( i \)'s online status at \( t = T \): is 1 if online, 0 if not online 

\( n_{t,i}^{\text{warmup}} \)  
Number of consecutive increments (inclusive) combustion turbine generator \( i \) has been warming up by \( t = T \) 

\( n_{t,i}^{\text{online}} \)  
Number of consecutive increments (inclusive) combustion turbine generator \( i \) has been online by \( t = T \) 

\( n_{t,i}^{\text{off}} \)  
Number of consecutive increments (inclusive) combustion turbine generator \( i \) has been off by \( t = T \) 

\( \phi_{t,i}^{\text{warmup}} \)  
Whether combustion turbine generator \( i \) does not satisfy the minimum warm-up time at \( t = T \): is 1 if minimum warm-up time is not yet satisfied, 0 otherwise 

\( \phi_{t,i}^{\text{online}} \)  
Whether combustion turbine generator \( i \) does not satisfy the minimum online time at \( t = T \): is 1 if minimum online time is not yet satisfied, 0 otherwise 

\( \phi_{t,i}^{\text{off}} \)  
Whether combustion turbine generator \( i \) does not satisfy the minimum off time at \( t = T \): is 1 if minimum off time is not yet satisfied, 0 otherwise 

\( p_{t,i} \)  
Combustion turbine generator \( i \)'s output (MW) at \( t = T \)
3.3 Model

Following Powell’s notation, the hour-ahead problem can be modeled by defining the state variable, decision variables, exogenous information, transition functions, and objective function (201). Each component of the model is described below.

3.3.1 State Variable

Powell defines the state variable as the “minimally dimensioned function of history that is necessary and sufficient to compute the decision function, the transition function, and the contribution function” (2010). In other words, the state variable consists of the least amount of information that is necessary and sufficient for the decision-making process.

Let $I^G$ be the set of all gas generators. Let $(x_{t,i})_i$ denote the vector of $x_{t,i}$ variables across all $i \in I^G$ for a given time $t$, where $x$ represents a generic variable that is indexed by both time and generator. Then the state variable $S_t$ at time $t$ for the hour-ahead model is the following:

$$S_t = \{(w_{t,i})_i, (u_{t,i})_i, (n_{t,i}^{\text{wrm}})_i, (n_{t,i}^{\text{on}})_i, (n_{t,i}^{\text{off}})_i, (\phi_{t,i}^{\text{wrm}})_i, (\phi_{t,i}^{\text{on}})_i, (\phi_{t,i}^{\text{off}})_i\}$$

Together, $w_{t,i}$ and $u_{t,i}$ determine the time $t$ continuous state of generator $i$: whether it is warming up, online, or off (if $w_{t,i}$ and $u_{t,i}$ are both 0, then the generator is off). Of the three variables $n_{t,i}^{\text{wrm}}$, $n_{t,i}^{\text{on}}$, and $n_{t,i}^{\text{off}}$, at most one is nonzero. Similarly, of the three variables $\phi_{t,i}^{\text{wrm}}$, $\phi_{t,i}^{\text{on}}$, and $\phi_{t,i}^{\text{off}}$, at most one is nonzero. Since it is not known in advance which is nonzero, the state variable must include all of these variables to guarantee sufficient information to compute the transition function.
When the hour-ahead model is integrated into the simulation, the state variable is written as $S^H_d h$, where day $d$ and hour $h$ of the simulation define the time $t$ when the hour-ahead model is called. Chapter 4 explores this notation in further detail.

### 3.3.2 Decision Variables

The decision variables are chosen at time $t$ for each time increment $t' = 1, \ldots, T$. They are chosen based on a particular policy $\pi$, which Powell defines as a “rule or function to determine a decision given the available information in the state $S_t$” (2010).

For convenience, the decision variables’ dependence on $\pi$ is not explicitly denoted.

For each time increment, there are six decision variables for each combustion turbine generator as well as one slack decision variable. These variables determine how much power each generator supplies at a given time:

- $w_{t,t',i} = \begin{cases} 1 & \text{if generator } i \text{ is warming up at time } t' \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, t' = 1, \ldots, T$

- $w_{t,t',i}^{on} = \begin{cases} 1 & \text{if generator } i \text{ begins warming up at time } t' \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, t' = 1, \ldots, T$

- $u_{t,t',i} = \begin{cases} 1 & \text{if generator } i \text{ is online at time } t' \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, t' = 1, \ldots, T$

- $y_{t,t',i}^{on} = \begin{cases} 1 & \text{if generator } i \text{ goes online at time } t' \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, t' = 1, \ldots, T$

- $y_{t,t',i}^{off} = \begin{cases} 1 & \text{if generator } i \text{ turns off at time } t' \\ 0 & \text{otherwise} \end{cases}, \forall i \in I, t' = 1, \ldots, T$

- $p_{t,t',i} \geq 0$, the output (MW) of generator $i$ at time $t'$, $\forall i \in I, t' = 1, \ldots, T$

- $\epsilon_{t,t'}$, the total slack generation at time $t'$, $\forall t' \in 1 \ldots T$
The decision variables are chosen by a decision function $X^\pi(S_t)$ that depends on policy $\pi$ and state variable $S_t$, where $t$ is the time that the function is called. The policy takes into account the tunable parameters and the algorithm used to solve the hour-ahead problem (in this case, mixed integer linear programming). The decision function outputs decision variables for all applicable $t' = 1, \ldots, T$ in the time horizon and $i \in I^G$. The relation can be expressed in matrix form as follows:

$$X^\pi(S_t) = \begin{bmatrix}
(w_{t,t',i}^o)_{t',i} & (w_{t,t',i}^o)_{t',i} & (u_{t,t',i})_{t',i} & (y_{t,t',i}^{on})_{t',i} & (y_{t,t',i}^{off})_{t',i} & (p_{t,t',i})_{t',i} & (e_{t,t',i})_{t'}
\end{bmatrix}
$$

$$= x_t$$

In the simulation model defined in Chapter 4, the hour-ahead model decision function is written as $X^{H,\pi}(\cdot)$, where $H$ denotes the hour-ahead model.

### 3.3.3 Exogenous Information

The random components of this problem are $\mathcal{D}_{t,t'}$ and $p_{i,t'}^W$, which are the load demanded at time $t' > t$ and the amount of wind power available at time $t' > t$, respectively. Each is random at present time $t$. These values are replaced by actual demand values $\mathcal{D}_{t}$ and actual wind values $p_{i,t'}^W$ may be substituted.

Let $P_{i,t'}^S$ be the current slow generation at time $t'$ from generators that cannot be adjusted on a sub-hourly basis. Then the effective actual demand that the hour-ahead model must satisfy is:

$$D_{t'} = \mathcal{D}_{t'} - (P_{i,t'}^S + p_{i,t'}^W)$$

As wind power and current slow generation increase, less power is needed to be generated by the hour-ahead model.
3.3.4 Transition Functions

The transition functions determine each component of the state variable for the next time increment $t + 1$. They govern how the system proceeds from sub-hourly time increment $t$ to increment $t + 1$:

$$S_{t+1} = S^M(S_t, X^\pi(S_t), \hat{W}_{t+1})$$

Here, $S^M(\cdot)$ is the transition function, which inputs the current state variable, the decision variables computed for the current time, and the exogenous information $\hat{W}_{t+1}$. It outputs the state variable of the next time period.

Many individual transition functions make up $S^M(\cdot)$. The intra-hour transition functions hold for times $t = 1, \ldots, T - 1$ and all generators $i \in I^G$. They determine the state variable components for times $t + 1 = 2, \ldots, T$.

The inter-hour transition functions, which determine the value of the state variable at time $t = 1$, are obtained from the intra-hour transition functions through a simple notational modification: every variable with original subscript $t$ is changed to have subscript $T$, and every variable with original subscript $t + 1$ is changed to have subscript $t$. The inter-hour transition functions hold only for $t = 1, i \in I^G$. This notational modification is used for the mathematical model because no memory constraints are binding. In the actual code, the intra-hour and inter-hour transition functions are written distinctly, which is explained later in the section on constraints.

Within each hour, the transition variables below are calculated iteratively for $t = 1, \ldots, T$, but they are used in the code for inter-hour transition only at time $t = T$. The intra-hour transition functions are below:
1. **Online status transition function:**

\[ u_{t+1,i} = \begin{cases} 
1 & \text{if } y_{t+1,i}^{on} = 1 \\
0 & \text{if } (y_{t+1,i}^{off} = 1) \text{ or if } (w_{t+1,i}^{on} = 1) \\
u_{t,i} & \text{otherwise}
\end{cases} \]

The first case is when generator \( i \) goes online at the instant \( t + 1 \). In this implementation, the convention is that continuous status variables \( w \) and \( u \) become 1 during the same time increment that their instantaneous counterparts \( w_{t+1,i}^{on} \) and \( y_{t+1,i}^{on} \) become 1. The second case is when generator \( i \) turns off or begins warming up at the instant \( t + 1 \). For all other cases, the online state of generator \( i \) does not change.

2. **Warming up status transition function:**

\[ w_{t+1,i} = \begin{cases} 
1 & \text{if } w_{t+1,i}^{on} = 1 \\
0 & \text{if } (y_{t+1,i}^{on} = 1) \text{ or if } (y_{t+1,i}^{off} = 1) \\
w_{t,i} & \text{otherwise}
\end{cases} \]

The first case is when generator \( i \) begins warming up at the instant \( t + 1 \). The second case is when generator \( i \) goes online or turns off at the instant \( t + 1 \). For all other cases, the warming up state of generator \( i \) does not change.

3. **Consecutive warm-up time transition function:**

\[ n_{t+1,i}^{warmup} = \begin{cases} 
1 & \text{if } w_{t+1,i}^{on} = 1 \\
0 & \text{if } (u_{t+1,i} = 1) \text{ or if } (u_{t+1,i} = 0 \text{ and } w_{t+1,i}^{on} = 0) \\
n_{t,i}^{warmup} + 1 & \text{if } w_{t+1,i} = 1
\end{cases} \]

The first case is when generator \( i \) begins warming up at the instant \( t + 1 \). The second case is separated into two subcases: if generator \( i \) is already online at
$t + 1$, or if it is already off at $t + 1$, then the number of consecutive warm-up time increments is set to 0. Otherwise, if generator $i$ continues to be in a warming up state at $t + 1$, it is incremented by one.

4. **Consecutive online time transition function:**

\[
n_{t+1,i}^{\text{online}} = \begin{cases} 
1 & \text{if } y_{t+1,i}^{\text{on}} = 1 \\
0 & \text{if } (w_{t+1,i} = 1) \text{ or if } (u_{t+1,i} = 0 \text{ and } w_{t+1,i} = 0) \\
n_{t,i}^{\text{online}} + 1 & \text{if } u_{t+1,i} = 1 
\end{cases}
\]

The first case is when generator $i$ goes online at the instant $t + 1$. The second case is separated into two subcases: if generator $i$ is already warming up at $t + 1$, or if it is already off at $t + 1$, then the number of consecutive online time increments is set to 0. Otherwise, if generator $i$ continues to be online at $t + 1$, it is incremented by one.

5. **Consecutive off time transition function:**

\[
n_{t+1,i}^{\text{off}} = \begin{cases} 
1 & \text{if } y_{t+1,i}^{\text{off}} = 1 \\
0 & \text{if } (u_{t+1,i} = 1) \text{ or if } (w_{t+1,i} = 1) \\
n_{t,i}^{\text{off}} + 1 & \text{if } (w_{t+1,i} = 0 \text{ and } u_{t+1,i} = 0) 
\end{cases}
\]

The first case is when generator $i$ turns off at the instant $t + 1$. The second case is separated into two subcases: if generator $i$ is already online at $t + 1$, or if it is already warming up at $t + 1$, then the number of consecutive off state time increments is set to 0. Otherwise, if generator $i$ continues to be off at $t + 1$, it is incremented by one.
6. Minimum warm-up time satisfaction transition function:

\[
\phi_{t+1,i}^{\text{warmup}} = \begin{cases} 
0 & \text{if } (n_{t,i}^{\text{warmup}} = \tau_i^{\text{warmup}} \text{ and } w_{t,i} = 1) \\
1 & \text{if } (n_{t,i}^{\text{warmup}} < \tau_i^{\text{warmup}} \text{ and } (w_{t,i} = 1 \text{ or } w_{t+1,i}^{\text{on}} = 1)) \\
\phi_{t,i}^{\text{warmup}} & \text{if } (w_{t,i} = 0 \text{ and } w_{t+1,i}^{\text{on}} = 0)
\end{cases}
\]

The first case is when generator \(i\) completed the last increment of warming up during the last time increment, so the minimum warm-up time is already satisfied by this time increment. The second case is when generator \(i\) did not complete its last increment of warming up during the last time increment or has just begun to warm up, so the minimum warm-up time is not satisfied during this time increment. Otherwise, the satisfaction indicator stays the same if the generator has not been warming up since the last time increment.

7. Minimum online time satisfaction transition function:

\[
\phi_{t+1,i}^{\text{on}} = \begin{cases} 
0 & \text{if } (n_{t,i}^{\text{online}} \geq \tau_i^{\text{online}} \text{ and } u_{t,i} = 1) \\
1 & \text{if } (n_{t,i}^{\text{online}} < \tau_i^{\text{online}} \text{ and } (u_{t,i} = 1 \text{ or } y_{t+1,i}^{\text{on}} = 1)) \\
\phi_{t,i}^{\text{on}} & \text{if } (u_{t,i} = 0 \text{ and } y_{t+1,i}^{\text{on}} = 0)
\end{cases}
\]

The first case is when generator \(i\) completed the last increment of its minimum online time during the last time increment, so the minimum online time is already satisfied by this time increment. The second case is when generator \(i\) did not finish its minimum online time during the last time increment or has just gone online, so the minimum online time is not satisfied during this time increment. Otherwise, the satisfaction indicator stays the same if the generator has not been online since the last time increment.
8. **Minimum off time satisfaction transition function:**

\[
\phi_{t+1,i}^{\text{off}} = \begin{cases} 
0 & \text{if } \left(n_{t,i}^{\text{off}} \geq \tau_{i}^{\text{off}} \text{ and } (w_{t,i} = 0 \text{ and } u_{t,i} = 0)\right) \\
1 & \text{if } \left(n_{t,i}^{\text{off}} < \tau_{i}^{\text{off}} \text{ and } \left((w_{t,i} = 0 \text{ and } u_{t,i} = 1) \text{ or } y_{t+1,i}^{\text{off}} = 1\right)\right) \\
\phi_{t,i}^{\text{off}} & \text{if } \left((w_{t,i} = 1 \text{ or } u_{t,i} = 1) \text{ and } y_{t+1,i}^{\text{off}} = 0\right)
\end{cases}
\]

The first case is when generator \(i\) completed the last increment of its minimum off time during the last time increment, so the minimum off time is already satisfied by this time increment. The second case is when generator \(i\) did not finish its minimum off time during the last time increment or has just turned off, so the minimum off time is not satisfied during this time increment. Otherwise, the satisfaction indicator stays the same if the generator has not been off since the last time increment.

9. **Going online instantaneous indicator transition function:**

\[
y_{t+1,i}^{\text{on}} = \begin{cases} 
1 & \text{if } \left(\phi_{t+1,i}^{\text{warmup}} = 0 \text{ and } w_{t,i} = 1\right) \\
0 & \text{otherwise}
\end{cases}
\]

The only case when generator \(i\) can go online is when it satisfied the minimum warm-up time in the previous time increment. Furthermore, generator \(i\) must go online when it satisfied the minimum warm-up time in the previous time increment.

Of the three instantaneous indicator variables \(\{w_{t,i}^{\text{on}}, y_{t,i}^{\text{on}}, y_{t,i}^{\text{off}}\}\), \(y_{t,i}^{\text{on}}\) is the only one that is governed by a transition constraint. The other two variables are prevented from equaling 1 in certain situations but cannot be forced to equal 1 (e.g. the begin warm-up instantaneous variable can be 1 only if the minimum off time has already been satisfied, but the converse is not true).
These two material conditional requirements can be expressed as the following:

\[
w_{t+1,i}^{on} = 1 \rightarrow (\phi_{t+1,i}^{off} = 0 \text{ and } (w_{t,i} = 0 \text{ and } u_{t,i} = 0))
\]

\[
y_{t+1,i}^{off} = 1 \rightarrow (\phi_{t+1,i}^{online} = 0 \text{ and } u_{t,i} = 1)
\]

### 3.3.5 Objective Function

The objective function is usually written as the minimization over policies \(\pi\) of an expected sum of contribution functions, which are functions of the policies:

\[
\min_{\pi} \mathbb{E} \sum_{t} C(S_t, x^\pi(S_t))
\]

For the hour-ahead unit commitment problem, the objective is to minimize the expected total system cost of power generation. The contribution functions, therefore, are the fuel costs and penalties for not providing sufficient power. The contribution function for each generator \(i\) at time \(t\) is written as follows:

\[
C_{t,i}(S_t, x^\pi(S_t)) = C_{t,i}^{fuel} \times p_{t,i} + C_{t}^{shortage} \times \epsilon_t
\]

Here, \(p_{t,i}\) and \(\epsilon_t\) represent decision variables chosen based on policy \(\pi\). They can be written as \(p_{t,i}^\pi\) and \(\epsilon_t^\pi\) to make the dependence explicit. Since the total load to be satisfied at each time is random, and since different sample paths lead to different contributions, the goal is to minimize expected total system cost, which is a summation across all generators and time periods.

Define \(\theta\) as the vector of tunable parameters for this simulation, which includes the algorithm for solving the hour-ahead problem, the horizon of the algorithm, and heuristics for increasing the horizon (these concepts are explained in detail in later
chapters). The vector $\theta$, therefore, is associated with a policy $\pi$, and the objective function can be written as the following:

$$F^\pi(\theta) = \mathbb{E} \sum_{t \in I} \sum_{t \in 1 \ldots T} C_{t,i}(S_t, x_\pi(S_t))$$

The minimization problem is minimized over policies and is written as $\min_\pi F^\pi(\theta)$. Finally, the minimization of the objective function is represented as:

$$\min_\pi \mathbb{E} \left[ \sum_{t \in I} \sum_{t \in 1 \ldots T} \left( C_{t,i}^{\text{fuel}} \times p_{t,i}^\pi + C_{t}^{\text{shortage}} \times \epsilon_t^\pi \right) \right]$$

### 3.4 Constraints

This thesis solves the hour-ahead unit commitment problem as a mixed integer linear program, which requires writing the hour-ahead model – in particular, the transition functions and assumptions – as linear equality and inequality constraints.

For the hour-ahead model’s notation, time ranges from 1 to $T$, inclusive, where $T$ is the number of time increments per hour. In the simulation model’s notation, which is defined in Chapter 4, time ranges from 0 to $T - 1$, inclusive. This difference arises because the simulation model is dependent on its implementation in code, which follows the convention of indexing starting from zero, whereas the hour-ahead model is defined independent of code, so it is more intuitive to index starting from one.

These constraints are separated into four types. The first type is the upper and lower bound constraints. The other three types deal with hour-to-hour transition. For the very first hour of the very first day of simulation, the intra-hour constraints and the initial
hour constraints hold. For all other hours of the first day, as well as all hours of every other day, the intra-hour and inter-hour transition constraints hold.

### 3.4.1 Upper and Lower Bound Constraints

These are the upper and lower bound constraints for all decision variables:

1. **Warming up status indicator bound constraints:**

   \[ w_{t,t',i} \leq 1 \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]
   \[ 0 \leq w_{t,t',i} \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]

2. **Begin warming up indicator bound constraints:**

   \[ w_{on}^{t,t',i} \leq 1 \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]
   \[ 0 \leq w_{on}^{t,t',i} \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]

3. **Online status indicator bound constraints:**

   \[ u_{t,t',i} \leq 1 \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]
   \[ 0 \leq u_{t,t',i} \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]

4. **Go online indicator bound constraints:**

   \[ y_{on}^{t,t',i} \leq 1 \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]
   \[ 0 \leq y_{on}^{t,t',i} \quad , \forall \ i \in I^G, t' = 1, \ldots, T \]
5. Turn off indicator bound constraints:

\[ y_{t,t',i}^{off} \leq 1, \forall i \in I^G, t' = 1, \ldots, T \]

\[ 0 \leq y_{t,t',i}^{off}, \forall i \in I^G, t' = 1, \ldots, T \]

6. Output bound constraints:

\[ p_{t,t',i} \leq p_i^{max}, \forall i \in I^G, t' = 1, \ldots, T \]

\[ p_i^{min} \leq p_{t,t',i}, \forall i \in I^G, t' = 1, \ldots, T \]

7. Slack generation bound constraints:

\[ \epsilon_{t,t'} \leq \infty, \forall t' = 1, \ldots, T \]

\[ 0 \leq \epsilon_{t,t'}, \forall t' = 1, \ldots, T \]

3.4.2 Intra-Hour Constraints

The following constraints hold for all combustion turbines at time increments within the hour. They can be divided into eight categories:

1. Generator capacity constraints:

\[ p_{t,t',i} \geq p_i^{min} \times u_{t,t',i}, \forall i \in I^G, t' = 1, \ldots, T \]

\[ p_{t,t',i} \leq p_i^{max} \times u_{t,t',i}, \forall i \in I^G, t' = 1, \ldots, T \]

These two constraints ensure that each generator, when online, produces within its minimum and maximum capacity limits, respectively. Conversely, when \( u_{t,t',i} = 0 \) and the generator is not online, the constraints force \( p_{t,t',i} = 0 \) since the generator has zero output.
2. **Generator warming up/online/off status constraints:**

\[
\begin{align*}
    w_{t,t',i}^{on} + y_{t,t',i}^{on} + y_{t,t',i}^{off} & \leq 1, \quad \forall i \in I^G, t' = 1, \ldots, T \\
    w_{t,t',i} + u_{t,t',i} & \leq 1, \quad \forall i \in I^G, t' = 1, \ldots, T
\end{align*}
\]

The first constraint ensures that at any time \(t'\), each generator can be in no more than one of the following instantaneous states: begin warming up, going online, and turning off. The generator can, of course, be in none of these three states.

The second constraint ensures that at any time \(t'\), each generator cannot be simultaneously warming up or online. The generator can of course be in neither state, namely the off state.

3. **Generator warming up/online/off state transition constraints:**

\[
\begin{align*}
    w_{t,t',i} - w_{t,t'-1,i} &= w_{t,t',i} - y_{t,t',i}^{on}, \quad \forall i \in I^G, t' = 2, \ldots, T \\
    u_{t,t',i} - u_{t,t'-1,i} &= y_{t,t',i}^{on} - y_{t,t',i}^{off}, \quad \forall i \in I^G, t' = 2, \ldots, T \\
    (1 - w_{t,t',i} - u_{t,t',i}) - (1 - w_{t,t'-1,i} - u_{t,t'-1,i}) &= y_{t,t',i}^{off} - w_{t,t',i}^{on}, \quad \forall i \in I^G, t' = 2, \ldots, T
\end{align*}
\]

Together with the previous set of constraints, the first constraint ensures that a generator cannot both begin warming up and go online in a single five minute increment (i.e. \([t' - 1, t']\)). The second constraint ensures that a generator cannot both begin warming up and turn off in a single increment. The third constraint ensures that a generator cannot both go online and turn off in a single increment.

Taken together, these three constraints ensure that if a generator is warming up, the next state must be the online state. If a generator is online, the next state must
be the off state. If a generator is off, the next state must be the warming up state. The order of transitioning between these states cannot be violated.

Note that in the third constraint, \((1 - w_{t,t',i} - u_{t,t',i})\) is a variable indicating whether generator \(i\) is in the off state at time \(t'\). It is equal to 1 if generator \(i\) is neither warming up nor online at \(t'\) (i.e. \(w_{t,t',i} = 0\) and \(u_{t,t',i} = 0\)) and 0 otherwise.

4. Minimum warm-up/online/off time negative constraints:

\[
\begin{align*}
W_{t,t',i}^{on} + \sum_{t''=t'+1}^{\min(T,t'+\tau_i^{\text{warmup}}-1)} y_{t,t'',i}^{on} & \leq 1, \text{ if } \tau_i^{\text{warmup}} > 1, \forall i \in I^G, t' = 1, \ldots, T \\
Y_{t,t',i}^{on} + \sum_{t''=t'+1}^{\min(T,t'+\tau_i^{\text{online}}-1)} y_{t,t'',i}^{off} & \leq 1, \text{ if } \tau_i^{\text{online}} > 1, \forall i \in I^G, t' = 1, \ldots, T \\
Y_{t,t',i}^{off} + \sum_{t''=t'+1}^{\min(T,t'+\tau_i^{\text{off}}-1)} w_{t,t'',i}^{on} & \leq 1, \text{ if } \tau_i^{\text{off}} > 1, \forall i \in I^G, t' = 1, \ldots, T
\end{align*}
\]

The first constraint ensures that a generator cannot go online until its minimum warm-up time has been satisfied. The second constraint ensures that a generator cannot turn off until its minimum online time has been satisfied. The third constraint ensures that a generator cannot begin warming up until its minimum off time has been satisfied.

These constraints are binding only if the respective minimum time parameters exceed one time increment. Furthermore, each summation upper bound includes a \(-1\) term because the time increment at which generator \(i\) begins warming up
(i.e. \( t' \) for \( w_{t,t',i}^{on} = 1 \)) is assumed to be the first increment in which \( w_{t,t',i} = 1 \) and therefore counts towards the minimum warm-up time.

Finally, these constraints are negative because they prevent the next state from happening when the minimum time has not been satisfied, but they do not force the next state to happen when the minimum time has been satisfied.

5. **Maximum warm-up time positive constraint:**

\[
y_{t,t',i}^{on} = w_{t,t'-r_i^{warmup},i}^{on}, \forall i \in G, t' = (r_i^{warmup} + 1), ..., T
\]

This positive constraint ensures that generators that begin warming up within an hour go online exactly after the minimum warm-up time is satisfied. When the generator is warming up, furthermore, the generator cannot go online or begin warming up again. This constraint is geared towards generators that begin and terminate warming up within the same hour. The inter-hour transition version of this constraint handles the case where the warm-up time finishes after this hour.

6. **Ramping constraints:**

\[
p_{t,t',i} \leq p_{t,t'-1,i} + \Delta_i^{up} + y_{t,t',i}^{on} \times (p_i^{min} - \Delta_i^{up}), \forall i \in G, t' = 2, ..., T
\]

\[
p_{t,t',i} \geq p_{t,t'-1,i} + \Delta_i^{down} - y_{t,t',i}^{off} \times M, \forall i \in G, t' = 2, ..., T
\]

The first constraint ensures that the magnitude of each generator’s increase in power output over any single time step does not exceed the ramp-up rate \( \Delta_i^{up} > 0 \), except at the moment of going online, when the generator is allowed to produce instantly at its minimum output.
The second constraint ensures that the magnitude of each generator’s decrease in power output over any single time step does not exceed the absolute value of the ramp-down rate $\Delta_{\text{down}} < 0$, except at the moment of turning off, when the generator’s output must go instantly to zero.

7. Demand satisfaction constraint:

$$\sum_{i \in i^G} p_{t,t',i} \geq D_{t'}$$

, $\forall t' = 1, \ldots, T$

This constraint requires total system generation to satisfy actual demand for each time period. In the implementation of the linear program, slack variables $\epsilon_{t'}$ for $t' = 1, \ldots, T$ are used to prevent infeasibility in case demand cannot be satisfied. These slack variables represent the amount of shortages if they exist. In the implementation of the linear program, therefore, the constraint appears as follows:

$$\epsilon_{t'} + \sum_{i \in i^G} p_{t,t',i} \geq D_{t'}$$

, $\forall t' = 1, \ldots, T$

8. Reserve constraint:

$$\sum_{i \in i^G} p_{t,t',i} \leq \sum_{i \in i^G} p_{i,\text{max}} \left( \rho_{\text{reserve}} \times \max_{t' = 1, \ldots, T} D_{t'} \right)$$

, $\forall t' = 1, \ldots, T$

This constraint prevents total current generation from exceeding the difference between total potential generation and the reserve requirement, which is a percentage $\rho_{\text{reserve}}$ of peak demand forecasts. In the hour-ahead model, $\rho_{\text{reserve}} = 0$. 

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3.4.3 Inter-Hour Transition Constraints

The following constraints require end-of-hour transition variables that take into account the state of the system when transitioning from one hour to the next. They can be divided into four categories:

1. Generator warming up/online/off status constraints carried over from the previous hour:

\[
w_{t,t',i} - w_{t,i} = w_{t,t',i}^{on} - y_{t,t',i}^{on}, \forall i \in I^G, t' = 1
\]

\[
u_{t,t',i} - u_{t,i} = y_{t,t',i}^{on} - y_{t,t',i}^{off}, \forall i \in I^G, t' = 1
\]

\[
(1 - w_{t,t',i} - u_{t,t',i}) - (1 - w_{t,i} - u_{t,i}) = y_{t,t',i}^{off} - w_{t,t',i}^{on}, \forall i \in I^G, t' = 1
\]

These constraints apply for \( t' = 1 \), when \( t' - 1 = T \) from the previous hour.

Hence \( w_{t,i} \) and \( u_{t,i} \) are input parameters to the model (the warming up and online statuses from the previous hour).

These constraints ensure that in a single time increment, generator \( i \) cannot both begin warming up and go online, nor both go online and turn off, nor both turn off and begin warming up.

2. Minimum warm-up/online/off time negative constraints carried over from the previous hour:

\[
\phi_{t,i}^{warmup} + \sum_{t''=1}^{T_i^u} y_{t,t'',i}^{on} \leq 1,
\]

\[
if (T_i^u > 1, \tau_i^{warmup} > 1, \phi_{t,i}^{warmup} > 0), \forall i \in I^G, t' = T
\]
\[
\phi_{t, i}^{\text{online}} + \sum_{t''=1}^{\min(T, \tau_{i, t_i}^{\text{online}} - n_{t, i}^{\text{online}})} y_{t, t''}^{\text{off}} \\
\leq 1 \quad \text{if } (\tau_{i, t_i}^{\text{online}} > 1 \text{ and } \phi_{t, i}^{\text{online}} > 0), \forall i \in I, t' = T
\]

\[
\phi_{t, i}^{\text{off}} + \sum_{t''=1}^{\min(T, \tau_{i, t_i}^{\text{off}} - n_{t, i}^{\text{off}})} w_{t, t''}^{\text{on}} \\
\leq 1 \quad \text{if } (\tau_{i, t_i}^{\text{off}} > 1 \text{ and } \phi_{t, i}^{\text{off}} > 0), \forall i \in I, t' = T
\]

where \( T^{u}_i = \min(T + 1, \tau_{i}^{\text{warmup}} - n_{t, i}^{\text{warmup}}) \) is the upper bound time for generator \( i \).

These constraints ensure that each generator abides by minimum warm-up, online, and off times when transitioning to the next hour. They are binding only if the respective minimum times exceed one time increment and only if \( \phi_{t, i}^{\text{warmup}} = 1, \phi_{t, i}^{\text{online}} = 1, \) and \( \phi_{t, i}^{\text{off}} = 1 \), respectively; otherwise, the minimum times have already been satisfied.

There is an off-by-one term in the minimum function of the summation upper bound \( T^{u}_i \) for the first constraint. When \( T^{u}_i = 1 \), the generator is forced to go online during the first increment of the new hour, so this constraint is unneeded. The off-by-one term is used to satisfy the counterpart positive constraint explained below (the other two constraints have no counterpart positive constraints).
3. Maximum warm-up time positive constraint carried over from the previous hour:

$$y_{t,t',i}^{on} = 1, \text{ if } (T_i^u \leq T \text{ and } \tau_i^{warmup} > 1 \text{ and } \phi_i^{warmup} > 0), \forall i \in I^G, t' = T_i^u$$

where $$T_i^u = \min(T + 1, \tau_i^{warmup} - \phi_i^{warmup})$$ is the upper bound time for generator $$i$$.

The constraint ensures that the generator goes online exactly at the upper bound, after the minimum warm-up time is satisfied. This constraint holds only if the minimum warm-up time is greater than one time increment and it had not been satisfied during the previous hour ($$\phi_i^{warmup} > 0$$); otherwise, the generator would have gone online during the previous hour. In addition, the upper bound must be less than $$T + 1$$, which guarantees that the generator will go online within this hour. On the other hand, if the upper bound is equal to $$T + 1$$, the generator will go online in a later hour.

4. Ramping constraints carried over from the previous hour:

$$p_{t,t',i} \leq p_{t,i} + \Delta_i^{up} + y_{t,t',i}^{on} \times (p_i^{min} - \Delta_i^{up}) \quad , \forall i \in I^G, t' = 1$$

$$p_{t,t',i} \geq p_{t,i} + \Delta_i^{down} - y_{t,t',i}^{off} \times M \quad , \forall i \in I^G, t' = 1$$

These constraints hold for the transition to the next hour at $$t' = 1$$, ensuring that ramp-up and ramp-down rates are not violated between consecutive time increments unless generator $$i$$ goes online or is turned off at that exact moment.
3.4.4 Initial Hour Constraints

The following constraints are needed only for the very first hour of the very first day of simulation because they govern the initial states of the generators. They can be divided into three categories:

1. Generator already off initial constraint:

\[ y_{t,t',i}^{off} = 0 \]
\[ , \forall i \in I^G, t' = 1 \]

No generators turn off during the first time increment of the first hour of the first day of the simulation. All generators are already off prior to the simulation.

2. Generator cold start initial constraint:

\[ y_{t,t',i}^{on} = 0 \]
\[ , \forall i \in I^G, t' = 1, \ldots, \tau_i^{warmup} \]

This constraint ensures that generators start from a cold state and must warm up prior to going online. Assuming that the generator begins warming up during the first increment of the first hour of the first day of simulation, the generator cannot go online until after the minimum warm-up time is satisfied. Generators that begin warming up after the first increment are governed by the intra-hour and inter-hour warming up state transition constraint.

3. Generator online state transition initial constraint:

\[ u_{t,t',i} = y_{t,t',i}^{on} \]
\[ , \forall i \in I^G, t' = 1 \]

This constraint is analogous to the intra-hour constraint \( u_{t,t',i} - u_{t,t'-1,i} = y_{t,t',i}^{on} - y_{t,t',i}^{off} \) but with \( u_{t,t'-1,i} = 0 \) because generators are assumed not to have
been online prior to the simulation. Combined with the previous constraint, this constraint ensures that a generator is not producing power during the first increment of the first hour of the first day of simulation.

3.5 Proposal to Model Combined Cycle Generators

From the analysis in Chapter 2, combined cycle generators may be viewed as a combination of two generators albeit with restrictions on operation. Below is a separability model for combined cycle generators, where each is modeled as two distinct generators – a gas turbine generator and a steam turbine generator – with certain dependencies between the variables governed by linear constraints. It is intended as a reference for future research when sufficient data is available.

3.5.1 Assumptions for Combined Cycle Generators

The following assumptions allow a combined cycle generator to be modeled as two distinct yet interdependent components:

1. Multi-shaft combined cycle generators are still separated into two components:
   Take a 4x1 combined cycle generator, which has four gas turbines and one steam turbine. Then all four gas turbines are combined as the gas turbine component, such that the component’s ramp rates as well as minimum and maximum outputs are each the sum of the individual parameters.

2. The gas and steam turbine components ramp independently:
   The only exception is that the steam turbine component cannot generate power when the gas turbine component is not online. If they are both online, however,
each individual output is governed only by its own ramp rate and capacity constraints.

3. Steam turbine component does not have a maximum or minimum output requirement before it can turn off:

   The time at which the steam turbine component can turn off is independent of its current output.

4. Available generator parameter data for each component:

   The data for the gas turbine and steam turbine components must be separate. The details are explained in the next subsection.

3.5.2 Variables and Parameters for Combined Cycle Generators

Let $\mathcal{G}^G$ be the set of all decision variables, generator parameters, and transition variables indexed for combustion turbine generator $i$ listed in Section 3.2 (thus, $e_{t,t'} \notin \mathcal{G}^G$ because it is not indexed by $i$). Define $\mathcal{G}^{CC}$ as the set of all variables and parameters associated with combined cycle generator $j$. For each member $g_i \in \mathcal{G}^G$, the same variable appears twice in $\mathcal{G}^{CC}$; denote them as $g_j$ and $g_j^*$. The former is the variable or parameter for the gas turbine component of combined cycle generator $j$, and the latter (denoted with superscript $^*$) is the corresponding variable or parameter for the steam turbine component of $j$. For example, since $w_{t,t'}, p_{t}^{min}$, and $w_{t,t'}$ are elements of $\mathcal{G}^G$, then $w_{t,t'}, p_{t,t'}^{min}, p_{j}^{min}, w_{t,t'}$, and $w_{t,t'}^*$ are elements of $\mathcal{G}^{CC}$. 
3.5.3 Constraints for Combined Cycle Generators

Let $I^{CC}$ be the set of combined cycle generators. Then each constraint that is associated with combustion turbine generator $i \in I^G$ and is listed in Section 3.4 appears *twice* for combined cycle generator $j \in I^{CC}$: once for the gas turbine component and once for the steam turbine component. In other words, combined cycle generators have analogous constraints for each component, with the difference that the set of steam turbine component constraints contains variables and parameters denoted by superscript $^\ast$.

In addition, each combined cycle generator $j \in I^{CC}$ has two additional constraints that govern the interdependence of its gas turbine and steam turbine components:

1. **Shutdown dependency constraint:**

   \[ y_{t,t'}^{\ast,off} = y_{t,t'}^{off}, \quad \forall j \in I^{CC}, t' = 1, \ldots, T \]

   This constraint ensures that the steam turbine component of combined cycle generator $j$ turns off when and only when the gas turbine component turns off.

2. **Boiling warm-up dependency constraint:**

   \[ w_{t,t'}^{\ast,on} = y_{t,t'}^{on}, \quad \forall j \in I^{CC}, t' = 1, \ldots, T \]

   This constraint ensures that the steam turbine component begins warming up when and only when the gas turbine component goes online (which is when it begins generating steam).
3.5.4 Parameter Constraints for Combined Cycle Generators

The additional constraints for combined cycle generators place restrictions on their minimum time parameters:

1. **Shutdown dependency parameter constraint:**
   \[ \tau_{j}^{\text{online}} \geq \tau_{j}^{\text{warmup}} + \tau_{j}^{\text{online}} \], \( \forall j \in I^{CC} \)

   The boiling warm-up constraint ensures that the gas turbine component’s time of going online always coincides with the steam turbine component’s time of beginning to warm up. If this parameter constraint is not satisfied, then the gas turbine component can turn off before the steam turbine component has satisfied its minimum off time, which would violate the shutdown dependency constraint.

2. **Boiling warm-up dependency parameter constraint:**
   \[ \tau_{j}^{\text{off}} + \tau_{j}^{\text{warmup}} \geq \tau_{j}^{*\text{off}} \], \( \forall j \in I^{CC} \)

   The shutdown dependency constraint ensures that the gas turbine and steam turbine components always begin their off states simultaneously. If this parameter constraint is not satisfied, then the gas turbine component can go online before the steam turbine component can begin warming up, which would violate the boiling warm-up dependency constraint.

   Certain minimum time parameters may need to be increased to satisfy the parameter constraints (increasing the minimum time parameters makes the generators worse off but preserves realism; decreasing them makes the generators better off and,
therefore, is prohibited). Define $\tau_{j}^{\text{online}}$ and $\tau_{j}^{\text{warmup}}$ as the gas turbine component’s modified minimum online and warm-up times, respectively:

$$
\tau_{j}^{\text{online}} = \max(\tau_{j}^{\text{online}}, \tau_{j}^{\text{warmup}} + \tau_{j}^{\text{online}}) \quad \forall j \in I_{CC}
$$

$$
\tau_{j}^{\text{warmup}} = \max(\tau_{j}^{\text{warmup}}, \tau_{j}^{\text{off}} - \tau_{j}^{\text{off}}) \quad \forall j \in I_{CC}
$$

Using these modified values and the other original parameters guarantees that the parameter constraints are satisfied. Alternatively, $\tau_{j}^{\text{off}} = \max(\tau_{j}^{\text{off}}, \tau_{j}^{\text{off}} - \tau_{j}^{\text{warmup}})$ and $\tau_{j}^{\text{warmup}}$ could be substituted for $\tau_{j}^{\text{warmup}}$ and $\tau_{j}^{\text{off}}$.

This separability model for combined cycle generators is intended to be a reference for further research when separate combined cycle component data is available. Including combined cycle generators into the set of fast generators that is scheduled by the hour-ahead model may provide insights on how to prevent shortages and reduce cost.
Chapter IV

4 The Simulation Model

Each simulation is run for 20 days, with the day-ahead model called once each day and the hour-ahead model called 24 times each day. For each day $d$, the day-ahead model creates a generation schedule for each hour, which gives the total planned power. The simulation then runs through each hour $h$ of day $d$ and implements the slow power from the day-ahead schedule. For this simulation model, slow power, or coal power, consists of the power generated by hydro, nuclear, and steam generators. Fast power, or gas power, consists of the power generated by combustion turbine, landfill, and diesel generators. Since actual wind and demand are given in hourly values, sub-hourly values must be simulated. This thesis divides up each hour into $T = 12$ sub-hourly increments of five minutes each. In the simulation model, the increment $s = 0$ is the first increment of each hour, and $s = T - 1$ is the last increment of each hour. Similarly, $d = 0$ is the first day of simulation and $h = 0$ is the first hour of each day.

After actual wind and slow power are subtracted from actual demand, the remaining sub-hourly values are the demand to be satisfied by the fast generators. These demand values are passed into the hour-ahead model, which solves a mixed integer linear program and creates a generation schedule of the fast generators only, for each five minute increment. Since these demand values are assumed to be the actual demand, the generation schedule of the hour-ahead model is implemented as the real-time simulation.
As a result, the statuses of the fast generators from the day-ahead solution are updated according to the hour-ahead solution.

After simulating each hour of day $d$, the simulation updates the total actual power supplied and calculates generation costs and shortage penalties. It then calls the day-ahead model for day $d + 1$ and repeats the process until each day and hour is simulated.

As such, the hour-ahead model described in the previous chapter accounts for only one component of the total simulation process. This chapter describes how the hour-ahead model fits in with the rest of the simulation and provides a mathematical model for how the simulation functions as a whole.

### 4.1 Assumptions from the Day-Ahead Model

Below are several important assumptions from the day-ahead model that contrast with assumptions of the hour-ahead model. The list is not exhaustive: the implementation of the day-ahead model is beyond the scope of this work and can be found in Kevin Kim’s senior thesis.

1. **Reserve requirement for the day-ahead model:**

   Unlike the hour-ahead model, the day-ahead model has a reserve requirement because its uses predicted demands. Following NERC guidelines, the reserve requirement percentage used in the day-ahead model is $\rho_{\text{reserve}} = 0.01$ times the maximum forecasted hourly demand for each day (Botterud et al., 2009).
2. **Day-ahead wind constraint applies to the total output of all wind generators:**

   Let $W_{t,t'}^P$ be the predicted wind for time $t'$. Then the total day-ahead planned wind generation cannot exceed $W_{t,t'}^P$:

   $$\sum_{t \in I^W} p_{t,t',l} \leq W_{t,t'}^P \quad \forall t' = 1, \ldots, T^d$$

   Here, $I^W$ is the set of wind generators and $T^d = 24$ is the number of time increments per day. Recall that the hour-ahead model does not deal with wind predictions or wind constraints because it subtracts actual wind from the total demand to be satisfied.

3. **Wind generators incur no marginal cost and do not contribute constraints to the day-ahead mixed integer linear program:**

   There are two reasons for this assumption. First, marginal cost calculations for wind may be complex when they accurately reflect the mechanics of wind farm operation and are outside the scope of the day-ahead model. Second, the original wind data is altered prior to each simulation to reflect the wind penetration for that simulation. Planned wind output from the day-ahead model must match the altered predicted wind data, and it is therefore not subject to constraints.

### 4.2 List of Variables

The following table lists the variables used in the simulation model. Since the hour-ahead model is a component of the simulation model, the list of hour-ahead variables is a subset of this list. The notation is different from that found in the hour-
ahead model to account for the difference between the day-ahead and hour-ahead models. Matrix notation is used through the bolding of variables, and time indices begin at zero. 

N.B.: Generator indices also begin at zero in the simulation model (instead of one in the hour-ahead model), but this difference is less crucial because the model usually refers to generators as a set rather than as individuals.

In addition, the decision variable notation used in the hour-ahead model is clarified. The expression \( a_{t,t',i} \) is defined in the hour-ahead model as the information or decision regarding generator \( i \) that is known or made at time \( t \) and is actionable or executed at time \( t' > t \). In the simulation, time is measured by a combination of days, hours, and sub-hourly increments, but it is possible to express time solely in hours or solely in sub-hourly increments using parallel time notation. Let

\[
f(d, h) = T^d \times d + h
\]

where \( T^d = 24 \) is the number of time increments per day and \( d = m, h = n \) indicate the \((n + 1)^{th}\) hour of the \((m + 1)^{th}\) day of the simulation. Then \( f(d, h) \) is the cumulative \((f(d, h) + 1)^{th}\) hour of the simulation. For example, \( f(0,0) = 0 \) is the first hour of simulation. Similarly, let

\[
f(d, h, s) = T \times f(d, h) + s
\]

\[
= T^d \times T \times d + T \times h + s
\]

where \( T = 12 \) is the number of time increments per hour and \( s = r \) indicates the \((r + 1)^{th}\) increment of the \((n + 1)^{th}\) hour of the \((m + 1)^{th}\) day of the simulation. Then \( f(d, h, s) \) is the cumulative \((f(d, h, s) + 1)^{th}\) increment of the simulation.

This notation clarifies the notation found in the previous chapter. The hour-ahead variable \( a_{t,t',i} = a_{f(d,h),f(d,h,s),i} \) is the information or decision regarding generator \( i \) that
is known or made by the hour-ahead model during the cumulative hour \( t = f(d, h) \) and is actionable or implemented at the cumulative time increment \( t' = f(d, h, s) > t \), i.e. the \((s + 1)^{th}\) sub-hourly increment of cumulative hour \( f(d, h) \).

**Table 4-1: List of Variables for Simulation Model**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Variables</td>
<td></td>
</tr>
<tr>
<td>( w_d^D )</td>
<td>Matrix of “warming up/not warming up” status indicator variables scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s indicator for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( w_d^{on,D} )</td>
<td>Matrix of “begin warming up/do not begin warming up” indicator variables scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s indicator for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( u_d^D )</td>
<td>Matrix of “is online/is not online” status indicator variables scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s indicator for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( y_d^{on,D} )</td>
<td>Matrix of “go online/do not go online” indicator variables scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s indicator for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( y_d^{off,D} )</td>
<td>Matrix of “turn off/do not turn off” indicator variables scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s indicator for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( p_d^D )</td>
<td>Matrix of committed generation scheduled by the day-ahead model on day ( d ); element ((h, i)) is generator ( i )'s output during hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( e_d^D )</td>
<td>Matrix of slack variables representing power shortages caused by the day-ahead model on day ( d ); element ((h)) is the total amount for hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>$w_{d,h}^{H}$</td>
<td>Matrix of “warming up/not warming up” status indicator variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s indicator for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$w_{d,h}^{on,H}$</td>
<td>Matrix of “begin warming up/do not begin warming up” indicator variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s indicator for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$u_{d,h}^{H}$</td>
<td>Matrix of “is online/is not online” status indicator variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s indicator for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$y_{d,h}^{on,H}$</td>
<td>Matrix of “go online/do not go online” indicator variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s indicator for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$y_{d,h}^{off,H}$</td>
<td>Matrix of “turn off/do not turn off” indicator variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s indicator for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$p_{d,h}^{H}$</td>
<td>Matrix of actual generation variables scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s,j)$ is fast generator $j$’s actual generation for sub-hourly increment $s$ during that hour</td>
</tr>
<tr>
<td>$c_{d,h}^{H}$</td>
<td>Matrix of slack variables representing power shortages scheduled by the hour-ahead model on hour $h$ of day $d$; element $(s)$ is the total amount for sub-hourly increment $s$ during that hour</td>
</tr>
</tbody>
</table>

**Simulation Parameters**

$D^s$ Number of days used in the simulation; set to 20
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^d$</td>
<td>Number of time increments per day used in the simulation; set to 24</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of time increments per hour used in the simulation; set to 12</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Wind penetration (%)</td>
</tr>
<tr>
<td>$C_{d,h,s}^{\text{shortage}}$</td>
<td>Penalty $\left(\frac{s}{\text{MW}}\right)$ for power shortage during increment $s$ of hour $h$ of day $d$</td>
</tr>
</tbody>
</table>

**Simulation Data Components**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Set of all generators</td>
</tr>
<tr>
<td>$I^G$</td>
<td>Set of all fast generators; planned by the day-ahead model and rescheduled by the hour-ahead model</td>
</tr>
<tr>
<td>$I^C$</td>
<td>Set of all slow generators; planned by the day-ahead model and implemented exactly</td>
</tr>
<tr>
<td>$I^W$</td>
<td>Set of all wind generators; planned by the day-ahead model and implemented according to actual wind by the hour-ahead model</td>
</tr>
<tr>
<td>$W^P$</td>
<td>Matrix of predicted wind values; element $(d, h)$ – alternatively, element $(f(d, h))$ – is the predicted wind value (MW) for hour $h$ of day $d$</td>
</tr>
<tr>
<td>$W^A$</td>
<td>Matrix of actual wind values; element $(d, h)$ – alternatively, element $(f(d, h))$ – is the actual wind value (MW) for hour $h$ of day $d$</td>
</tr>
<tr>
<td>$L^P$</td>
<td>Matrix of predicted total demand values; element $(d, h)$ is the predicted total demand value (MW) for hour $h$ of day $d$</td>
</tr>
<tr>
<td>$L^A$</td>
<td>Matrix of actual total demand values; element $(d, h)$ is the actual total demand value (MW) for hour $h$ of day $d$</td>
</tr>
</tbody>
</table>
### Matrix of generator parameters for all generators

\[ G \]

- \( G \) represents the matrix of generator parameters for all generators \( i \in I \).

### Fuel cost for slow generator

\[ C_{d,h,i}^{fuel,C} \]

- \( C_{d,h,i}^{fuel,C} \) is the fuel cost per MWh for slow generator \( i \in I^C \) during hour \( h \) of day \( d \).

### Fuel cost for fast generator

\[ C_{d,h,s,i}^{fuel,G} \]

- \( C_{d,h,s,i}^{fuel,G} \) is the fuel cost per MWh for fast generator \( i \in I^G \) during increment \( s \) of hour \( h \) of day \( d \).

### Exogenous Variables

- \( p_{d,h}^w \) is the vector of simulated sub-hourly wind values taken to be actual values during hour \( h \) of day \( d \).

### Transition Variables

- \( \sigma_d^D \) is the vector of end-of-day variables required for inter-day transition in the day-ahead model from day \( d \) to \( d + 1 \).

- \( \sigma_{d,h}^H \) is the vector of hour-ahead transition variables required for inter-hour transition in the hour-ahead model from cumulative hour \( f(d,h) \) to \( f(d,h) + 1 \).

- \( p_{d,h}^s \) is the total slow generation (MW) that is planned by the day-ahead model for hour \( h \) of day \( d \) and is implemented by the hour-ahead model for each increment of that hour.

- \( \lambda_{d,h}^H \) is the vector of sub-hourly demand values that must be satisfied by fast generators only; is input to the hour-ahead model and excludes slow generation and actual wind power.

- \( S_{d,h,i}^{y_{on}} \) is the set of all sub-hourly increments \( s \) such that the instantaneous indicator variable \( y_{f(d,h),f(d,h,s),i}^{on} \) is 1 in hour \( h \) of day \( d \); used to modify the day-ahead solution with the hour-ahead solution.
### 4.3 Model

The simulation model illustrates the interaction of the day-ahead model and the hour-ahead model to create a multi-day simulation of power generation. For the day-ahead model, time is measured in both days and hours: day \( d \in [0, D^d - 1] \) and hour \( h \in [0, T^d - 1] \). For the hour-ahead model, time is further subdivided into smaller time intervals: sub-hourly increment \( s \in [0, T - 1] \). Each time index begins at zero and ends

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{d,h,i}^{\text{off}} )</td>
<td>Set of all sub-hourly increments ( s ) such that the instantaneous indicator variable ( y_{f(d,h),f(d,h,s),i}^{\text{off}} ) is 1 in hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( S_{d,h,i}^{\text{on}} )</td>
<td>Set of all sub-hourly increments ( s ) such that the instantaneous indicator variable ( w_{f(d,h),f(d,h,s),i}^{\text{on}} ) is 1 in hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( \tau_{d,h,i}^{\text{on}} )</td>
<td>The latest time increment, if it exists, during which the instantaneous indicator variable ( y_{f(d,h),f(d,h,s),i}^{\text{on}} ) is 1 in hour ( h ) of day ( d ); used to modify the day-ahead solution with the hour-ahead solution</td>
</tr>
<tr>
<td>( \tau_{d,h,i}^{\text{off}} )</td>
<td>The latest time increment, if it exists, during which the instantaneous indicator variable ( y_{f(d,h),f(d,h,s),i}^{\text{off}} ) is 1 in hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( \tau_{d,h,i}^{\text{on}} )</td>
<td>The latest time increment, if it exists, during which the instantaneous indicator variable ( w_{f(d,h),f(d,h,s),i}^{\text{on}} ) is 1 in hour ( h ) of day ( d )</td>
</tr>
<tr>
<td>( \tau_{d,h,i} )</td>
<td>The latest time increment, if it exists, during which any of the three instantaneous indicator variables ( \left(y_{f(d,h),f(d,h,s),i}^{\text{on}}, y_{f(d,h),f(d,h,s),i}^{\text{off}}, w_{f(d,h),f(d,h,s),i}^{\text{on}}\right) ) is 1 in hour ( h ) of day ( d )</td>
</tr>
</tbody>
</table>
at one less than its corresponding upper limit. Hence, days range from 0 to 19, inclusive; hours range from 0 to 23, inclusive; increments range from 0 to 11, inclusive.

4.3.1 State Variable

The state variable at day $d$, hour $h$ is written as follows:

$$ S_{d,h} = \{S^D_d, S^H_{d,h}\} $$

where $S^D_d$ is the state variable for the day-ahead model for day $d$ and $S^H_{d,h}$ is the state variable for the hour-ahead model for day $d$, hour $h$.

The details of the day-ahead state variable $S^D_d$ are beyond the scope of this work and can be found in Kevin Kim’s senior thesis. It suffices to write:

$$ X^{D,\pi}(S^D_d) = [u^D_d, y^{\text{on},D}_d, y^{\text{off},D}_d, w^D_d, w^{\text{on},D}_d, p^D_d, e^D_d] $$

In other words, the implementation of the day-ahead algorithm $X^{D,\pi}(\cdot)$ takes in the day-ahead state variable $S^D_d$ and returns seven two-dimensional matrices, each of size $T^d \times |I| = 24 \times |I|$ (except for $e^D_d$, which is size $24 \times 1$), where $I$ is the set of all generators used in the simulation. Each policy $\pi$ defines an implementation of the day-ahead model. For convenience, let $X^{D}(\cdot)$ be a particular implementation.

These matrices contain the mixed integer linear programming solution to the day-ahead problem. For example, $e^D_d(h)$ is the slack variable for hour $h$ of day $d$, and $u^D_d(h, i)$ is generator $i$’s planned “online/not online” status for hour $h$ of day $d$.

The hour-ahead state variable $S^H_{d,h}$, on the other hand, is defined in the hour-ahead model of the previous chapter using time $t$ notation. Recall that it is defined as:

$$ S_t = \{(w_{t,i})_t, (u_{t,i})_t, (n^{\text{warmup}}_{t,i})_t, (n^{\text{online}}_{t,i})_t, (n^{\text{off}}_{t,i})_t, (\phi_{t,i}^{\text{warmup}})_t, (\phi_{t,i}^{\text{online}})_t, (\phi_{t,i}^{\text{off}})_t\} $$
There is a conceptual difference between the $f(d, h)$ time indexing, which is used in the simulation, and the time $t$ indexing, which is used in the hour-ahead model, where $t \in [1, T]$. A method is needed to combine these two approaches. Define $S_{d,h}$ as the state variable for cumulative time increment $x$ that is computed at time $y, y \in [1, T]$ of the current hour. Then $x$ measures cumulative time in the simulation model, and $y$ measures time in the hour-ahead model. The state variable can be expressed as:

$$S_{d,h}^H = S_{f(d,h,T-1)|T}$$

This expression is evaluated at time $t = T$ in the hour-ahead model (recall that the hour-ahead model indexes time starting from one). It is also clarified that for the purposes of the simulation, the parameters $(w_{t,i})$, $(u_{t,i})$, $(n_{t,i}^{\text{warmup}})$, $(n_{t,i}^{\text{online}})$, $(n_{t,i}^{\text{off}})$, $(\phi_{t,i}^{\text{warmup}})$, $(\phi_{t,i}^{\text{online}})$, and $(\phi_{t,i}^{\text{off}})$ are fully defined only at cumulative time increment $f(d, h, T - 1)$, or time increment $T - 1$ of cumulative hour $f(d, h)$. In the hour-ahead model, they are defined for all time increments because of the iterative method through which they are calculated by the hour-ahead transition functions.

### 4.3.2 Decision Variables

The decision variables $x_{d,h}$ consist of both the day-ahead decision variables $x^{D}_{d}$ and the hour-ahead decision variables $x^{H}_{d,h}$, which are defined as follows:

$$x^{D}_{d} = [u^{D}_{d} \ y^{on,D}_{d} \ y^{off,D}_{d} \ w^{D}_{d} \ w^{on,D}_{d} \ e^{D}_{d}]$$: day-ahead augmented matrix for day $d$ decision variables

$$x^{H}_{d,h} = [u^{H}_{d,h} \ y^{on,H}_{d,h} \ y^{off,H}_{d,h} \ w^{H}_{d,h} \ w^{on,H}_{d,h} \ e^{H}_{d,h}]$$: hour-ahead augmented matrix for day $d$, hour $h$ decision variables

Recall that they are obtained from decision functions that depend on policy $\pi$: 69
The decision variables of the simulation model, therefore, can be written as the following:

\[ x_d^D = X^{D,\pi}(S_d^D) \]
\[ x_{d,h}^H = X^{H,\pi}(S_{d,h}^H) \]

4.3.3 Exogenous Information

The exogenous information \( \bar{W} \) has two components. The first is \( p_{d,h}^W \), the simulated actual wind power in MW for day \( d \), hour \( h \). This is a \( T \times 1 \) vector, where \( p_{d,h}^W(s) \) is the simulated actual wind power for sub-hourly increment \( s \). This vector is exogenous to the day-ahead and hour-ahead models. The actual wind power data \( W^A \) is given in hourly increments per day, but the hour-ahead model requires sub-hourly values. Hence sub-hourly values are simulated using a function \( \Omega^\pi(\cdot) \) that depends on the policy \( \pi \) (e.g. linear interpolation or Brownian bridge simulation):

\[ p_{d,h}^W = \Omega^\pi(W^A, d, h) \]

The second piece of exogenous information is \( p_{d,h}^L \), the simulated total load for day \( d \), hour \( h \). This is a \( T \times 1 \) vector, where \( p_{d,h}^L(s) \) is the simulated total demand that must be satisfied for sub-hourly increment \( s \). This vector is exogenous to the day-ahead and hour-ahead models. The actual demand data \( L^A \) is given in hourly increments per day, but the hour-ahead model requires sub-hourly values. Hence sub-hourly values are simulated using a policy \( \pi \)-dependent function \( \Lambda^\pi(\cdot) \):

\[ p_{d,h}^L = \Lambda^\pi(L^A, d, h) \]
The method of linear interpolation is used to obtain all sub-hourly demand values for the simulations. Let \( dp^L \) be the difference between current hourly demand and the next hourly demand split into equal sub-hourly intervals.

\[
dp^L = \frac{L^A(f(d,h+1)) - L^A(f(d,h))}{T}
\]

Then the actual demand is computed as follows:

\[
p_{d,h}(s) = L^A(f(d,h)) + s \times dp^L \quad \forall s \in [0, T - 1]
\]

Linear interpolation is used for all hours except the very last hour of the very last day of simulation. The end value of the interpolation \( L^A(f(d, h + 1)) \) does not appear in the same vector as the beginning value \( L^A(f(d, h)) \) to avoid repeating values. Instead it appears as the beginning value of the vector of the next hour. Linear interpolation is also used to generate sub-hourly wind values for all simulations unless otherwise noted.

4.3.4 Transition Functions

These functions incorporate the modifications made by the hour-ahead model to the day-ahead generation schedule. They are necessary for the day-ahead model to compute the end-of-day transition variables, which are needed to make the new day-ahead schedule for the next day. Let \( d \) and \( h \) be the current day and hour indices, respectively, and let \( d' \) and \( h' \) be the day and hour indices of the next hour. Then \( S^M(\cdot) \) is the transition function of the simulation model such that:

\[
S_{d',h'} = S^M \left( S_{d,h} \{ X^{D,\pi}(S_d^D) X^{H,\pi}(S_{d,h}^H) \}, \bar{W}_{d',h'} \right)
\]

In particular, the hour-ahead model may change the statuses and output of the fast generators, so the transition functions must transmit this information to the day-ahead
model. To do so, it is necessary to calculate the current slow generation planned by the
day-ahead model:

\[ p_{d,h}^S = \sum_{i \in I_C} p_{d}^D(h, i) \]

The hour-ahead model implements the schedule for slow generators because slow
generators generally cannot complete a full off–warm-up–online cycle within the hour.
Next, define the total load that must be satisfied by the fast generators only:

\[ \lambda_{d,h}^H = p_{d,h}^L - p_{d,h}^S - p_{d,h}^W \]

This is a \( T \times 1 \) vector of demands in sub-hourly increments. Then the output of
the hour-ahead model \( X_{d,h}(\cdot) \) can be written as:

\[
X_{d,h}(S_{d,h}^H, \lambda_{d,h}^H) = x_{d,h}^H
\]

\[
= [u_{d,h}^H, y_{d,h}^{on,H}, y_{d,h}^{off,H}, w_{d,h}^H, w_{d,h}^{on,H}, p_{d,h}^H, e_{d,h}^H]^
\]

Again, it is convenient to write \( X^H(\cdot) \) for a particular implementation of the hour-
ahead model. Each of the output matrices is of size \( T \times |I^G| \) except for \( e_{d,h}^H \), which is a
\( T \times 1 \) vector. For example, \( w_{d,h}^H(s, i) \) denotes gas generator \( i \)'s “warming up/not
warming up” status during sub-hourly increment \( s \) of hour \( h \) of day \( d \).

Let \( M(\cdot) \) be a mapping of indices such that index \( i \) in \( I^G \subset I \) maps to index
\( M(i) \) in \( I \). After each hour \( h \) of the simulation, the transition functions require that the
gas generator statuses in the day-ahead solution matrices be updated with the newly
determined hour-ahead values:

\[
u_{d}(h, M(i)) = u_{d,h}^H(T - 1, i) \quad, \forall i \in I^G
\]

\[
w_{d}(h, M(i)) = w_{d,h}^H(T - 1, i) \quad, \forall i \in I^G
\]

\[
p_{d}(h, M(i)) = p_{d,h}^H(T - 1, i) \quad, \forall i \in I^G
\]
The “online/not online” status, the “warming up/not warming up” status, and the output of each gas generator at each hour, from the day-ahead model’s point of view, are simply their respective values at the last (i.e. \( s = T - 1 = 11 \)) sub-hourly increment of the hour, from the hour-ahead model’s point of view. The reason is that these three variables are continuous, so whichever values they take at the end of the hour should be used by the day-ahead model to calculate the generation schedule for the following day. The values that they take within the hour are irrelevant to the day-ahead model.

The transition functions for the instantaneous decision variables, however, are different because the most recent time those were nonzero may not have occurred during increment \( s = 11 \). Define the following sets of instantaneous hit times:

\[
\delta_{d,h,i}^{\text{on}} = \{ s \in \mathbb{Z}: s \in [0, T - 1], y_{d,h}^{\text{on,H}}(s, i) = 1 \} , \forall i \in I^G
\]

\[
\delta_{d,h,i}^{\text{off}} = \{ s \in \mathbb{Z}: s \in [0, T - 1], y_{d,h}^{\text{off,H}}(s, i) = 1 \} , \forall i \in I^G
\]

\[
\delta_{d,h,i}^{\text{w, on}} = \{ s \in \mathbb{Z}: s \in [0, T - 1], w_{d,h}^{\text{w,H}}(s, i) = 1 \} , \forall i \in I^G
\]

These sets contain all sub-hourly increments within hour \( h \) during which the instantaneous decision variables are 1. Define the following transition times:

\[
\tau_{d,h,i}^{\text{on}} = \begin{cases} 
\max_{-\infty} \delta_{d,h,i}^{\text{on}} & \text{if } \delta_{d,h,i}^{\text{on}} \neq \emptyset \\
-\infty & \text{otherwise}
\end{cases}, \forall i \in I^G
\]

\[
\tau_{d,h,i}^{\text{off}} = \begin{cases} 
\max_{-\infty} \delta_{d,h,i}^{\text{off}} & \text{if } \delta_{d,h,i}^{\text{off}} \neq \emptyset \\
-\infty & \text{otherwise}
\end{cases}, \forall i \in I^G
\]

\[
\tau_{d,h,i}^{\text{w, on}} = \begin{cases} 
\max_{-\infty} \delta_{d,h,i}^{\text{w, on}} & \text{if } \delta_{d,h,i}^{\text{w, on}} \neq \emptyset \\
-\infty & \text{otherwise}
\end{cases}, \forall i \in I^G
\]

\[
\tau_{d,h,i} = \max (\tau_{d,h,i}^{\text{on}}, \tau_{d,h,i}^{\text{off}}, \tau_{d,h,i}^{\text{w, on}}, -1), \forall i \in I^G
\]
The first three transition times are the latest sub-hourly increments of hour \( h \) during which each instantaneous variable was 1 (if the instantaneous variable was 0 during the entire hour, then the value is set to \(-\infty\)). Transition time \( \tau_{d,h,i} \) is the latest sub-hourly increment of hour \( h \) where any of the three instantaneous decision variables was 1 (otherwise its value is \(-1\)). Then the transition functions are:

\[
\begin{align*}
y_{d}^{\text{on},D}(h, M(i)) &= \begin{cases} 
1 & \text{if } \tau_{d,h,i} = \tau_{d,h,i}^{\text{on}}, \\
0 & \text{otherwise} \end{cases}, \quad \forall \ i \in I^G \\
y_{d}^{\text{off},D}(h, M(i)) &= \begin{cases} 
1 & \text{if } \tau_{d,h,i} = \tau_{d,h,i}^{\text{off}}, \\
0 & \text{otherwise} \end{cases}, \quad \forall \ i \in I^G \\
w_{d}^{\text{on},D}(h, M(i)) &= \begin{cases} 
1 & \text{if } \tau_{d,h,i} = \tau_{d,h,i}^{\text{on}}, \\
0 & \text{otherwise} \end{cases}, \quad \forall \ i \in I^G
\end{align*}
\]

At any given hour \( h \), therefore, at most one of the instantaneous decision variables stored in the day-ahead solution has value 1. These functions give the day-ahead model the most up-to-date information to continue the simulation for the next day.

### 4.3.5 Objective Function

The objective function of the simulation model consists of minimizing the expected sum of the day-ahead slow generation costs and the hour-ahead fast generation costs and shortage penalties. The minimization is done over policies associated with tunable parameters. The decision variable vectors \( p_{d}^{D}, p_{d,h}^{H}, \) and \( e_{d,h}^{H} \) are dependent on \( \pi \) through the implementations of the day-ahead and hour-ahead models \( X_{d}^{D,\pi} \) and \( X_{d,h}^{H,\pi} \), respectively. Using \( p_{d}^{D,\pi}, p_{d,h}^{H,\pi}, \) and \( e_{d,h}^{H,\pi} \) to make the policy dependency explicit, the objective function can be represented as follows:
For the simulation model, costs for slow generators are ideally given per hour, whereas costs for fast generators and shortage penalties are ideally given per sub-hourly increment. Due to lack of data, however, these parameters may only be given per day.

In addition, the total cost for each simulation analyzed in the results does not include the costs from day 0 because of the time it takes for slow generators to get up to speed. For consistency, cost of fast generation and shortage penalties from day 0 are also not included.
Chapter V

5 Simulation Data

The simulation data are obtained from PJM and can be split into three categories: generator data, demand data, and wind data. Table 5-1 displays the distribution of generators used in the simulations.

Table 5-1: Generator Distribution in Simulations

<table>
<thead>
<tr>
<th>Type of Generator</th>
<th>Number Used in Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam</td>
<td>384</td>
</tr>
<tr>
<td>Nuclear</td>
<td>31</td>
</tr>
<tr>
<td>Hydro</td>
<td>77</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>482</td>
</tr>
<tr>
<td>Diesel</td>
<td>39</td>
</tr>
<tr>
<td>Landfill</td>
<td>28</td>
</tr>
<tr>
<td>Wind</td>
<td>12</td>
</tr>
</tbody>
</table>

The set of fast generators $I^G$ that is scheduled by the hour-ahead model contains 605 generators. It includes all of the combustion turbine, diesel, and landfill generators as well as the steam generators that are listed as having no minimum warm-up time.

One weakness of the PJM generator data is that generator costs are bid costs: the cost each generator claims to incur when submitting bids to PJM. The bid cost is not necessarily the generator’s actual cost of operation; for example, operators of coal generators with high fixed costs may bid zero cost when their generators are already online because selling output at any price would be more profitable than turning off the generator. Despite this inconsistency, the use of bid costs does not present a significant
problem because the total generation cost calculated at the end of the simulation is only an indicator of the simulation’s performance and should not be taken as a precise value.

5.1 Generator Data

PJM provides data on many parameters for each generator in its system, a few of which are used in the simulation. For each generator $i$, the necessary parameters are $\{p_i^{\text{min}}, p_i^{\text{max}}, \tau_i^{\text{online}}, \tau_i^{\text{off}}, \tau_i^{\text{warmup}}, \Delta_i^{\text{up}}, \Delta_i^{\text{down}}, C_i^{\text{fuel}}\}$. Each parameter is also listed below in uppercase and parentheses as it is categorized in the PJM data.

The minimum capacity $p_i^{\text{min}}$ is set to the economic minimum watts (ECONOMIC_MIN), given in MW. The maximum capacity $p_i^{\text{max}}$ is set to the economic maximum watts (ECONOMIC_MAX), given in MW. Values in MW do not need to be converted in the hour-ahead model because MW is a unit of power.

The minimum online time $\tau_i^{\text{online}}$ is set to the minimum run time (MIN_RUN_TIME). The minimum off time $\tau_i^{\text{off}}$ is set to the minimum down time (MIN_DOWN_TIME). The minimum warm-up time $\tau_i^{\text{warmup}}$ is set to the cold start-up time (COLD_STARTUP_TIME). PJM gives these three minimum times in hours, so they must be converted to number of five minute increments by multiplying by $T = 12$. In addition, each minimum time has a floor of one due to the implementation of the hour-ahead model.

The ramp-up rate $\Delta_i^{\text{up}}$ is set to the ramp rate (RAMP_RATE). The ramp-down rate $\Delta_i^{\text{down}}$ is set to negative the value of the ramp rate. PJM gives these rates in $\frac{\text{MW}}{\text{hr}}$, so they must be converted to $\frac{\text{MW}}{5 \text{ min}}$ by dividing by $T = 12$. 

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The fuel cost $C_{i,t}^{fuel}$ is given by a mean value of the cost curve as a function of the demand bids. In this implementation, therefore, it is not a function of time and can be written as $C_i^{fuel}$. The mean value is calculated from the minimum capacity lower bound to the maximum capacity upper bound. Let $C_i(x_t)$ be generator $i$’s cost in dollars given the generator’s bid $x_t$ at time $t$. Then the fuel cost is defined as follows:

$$C_i^{fuel} = \frac{\int_{p_i^{min}}^{p_i^{max}} C_i(x_t)dx_t}{p_i^{max} - p_i^{min}}$$

The integral is discretized and approximated with the trapezoidal rule by using price (PRICE1, PRICE2, PRICE3, etc.) and megawatt (MW1, MW2, MW3, etc.) values.

Within the mixed integer linear programming implementation, the shortage penalty is fixed for all time periods at $C^{shortage} = \frac{5000000000}{MW}$. When calculating the total simulation cost, however, the value is fixed for all time periods at $C^{shortage} = \frac{1000000}{MW}$. A larger value is used in the linear program to force demands to be satisfied whenever possible.

### 5.2 Demand Data

The demand data used is 2009 actual and predicted demand obtained from PJM. The dataset contains hourly values for predicted and actual demand for each day in 2009. For these 20 day simulations, the first day of each simulation is January 1, 2009.

Figure 5-1 is a graph of total actual demand versus total predicted demand during the 20 day simulation. Peak actual demand values tend to be under-predicted. As a
result, the hour-ahead model must reschedule the day-ahead solution to satisfy this demand.

![Actual Demand vs. Predicted Demand](image1)

**Figure 5-1: Actual Demand vs. Predicted Demand**

Figure 5-2 is a zoomed-in version of the same graph for hours 13 through 20 (day 0) of the data. Predicted demand is a step function because only hourly values are given; actual demand is not because sub-hourly values are simulated using linear interpolation.

![Actual Demand vs. Predicted Demand (Zoomed)](image2)

**Figure 5-2: Actual Demand vs. Predicted Demand (Zoomed)**
5.3 Wind Data

The simulation uses PJM’s actual wind data for each hour of each day in 2009. The predicted wind value for a given day and hour is the 7-day previous actual wind value measured at the same hour. For example, the predicted wind value for hour 15 on January 10, 2009 is the actual wind value for hour 15 on January 3, 2009. All wind values are initially scaled to simulate the effect of having more wind power available to the system.

The wind penetration $\omega^\pi$ is a simulation parameter that expresses the ratio of total actual wind power to total actual power. The parameter depends on policy $\pi$ but can be written as $\omega$ for convenience. Let $R$ be the matrix of reserve requirements calculated by the day-ahead model, where $R(d, h)$ is the reserve requirement for hour $h$ of day $d$. Then $\omega$ is calculated as follows:

$$\omega = \frac{\sum_d \sum_h \sum_s p^w_{d,h}(s)}{\sum_d \sum_h \sum_s (p^w_{d,h}(s) + R(d, h)) + \sum_{i \in \mathbb{Z}} \sum_d \sum_h \sum_s p^\pi_{d,i}(h, i) + \sum_{i \in \mathbb{Z}} \sum_d \sum_h \sum_s p^\pi_{d,h}(s, i)}$$

The summations are carried out over all days, all hours, and all sub-hourly increments of the simulation. The reserve requirement is incorporated because the wind penetration includes the day-ahead model’s perspective. Different levels of wind penetration are tested to analyze how well the system can integrate wind power. Since $\omega$ can be calculated only after the simulation terminates, however, several simulations are run to adjust the wind data to the appropriate scale so that the final wind penetration parameter matches the intended wind penetration of the simulation.
Figure 5-3 is a graph of actual wind versus predicted wind at the 5% level during the 20 day simulation. Wind is much more volatile than demand: the difference between actual and predicted wind is much larger than that of demand.

Figure 5-3: Actual Wind vs. Predicted Wind

Figure 5-4 is a zoomed-in version of the same graph for hours 13 through 20 (day 0) of the data. Again, the hourly predicted wind is a step function, whereas sub-hourly actual wind values are generated using linear interpolation.

Figure 5-4: Actual Wind vs. Predicted Wind (Zoomed)
Chapter VI

6 Results and Analysis

For each of the following cases, a 20 day simulation is run using linearly interpolated wind and demand values. Three levels of wind penetration are tested: \( \omega = 5.2\% \), \( \omega = 20.4\% \), and \( \omega = 40.0\% \). These simulations reveal substantial differences in the distribution of slow versus fast power, the number of shortages and overages, the distribution of total cost, and generator activity.

N.B.: The analyzed percentage differences and costs exclude day 0 of the simulation because it takes time for the slow generators to get up to speed.

6.1 5.2% Wind

The first simulation is run for 20 days with 5.2% wind penetration. The total cost of simulation is \$3.96 \times 10^{10}\$. The total cost is computed by summing generation costs and shortage penalties at each sub-hourly increment, excluding day 0. It is not directly comparable, therefore, to the total cost of a simulation that does not implement the hour-ahead model in sub-hourly increments.

6.1.1 The Advantages of Hour-Ahead Rescheduling

Figure 6-1 shows the planned power versus the predicted demand for the 20 day simulation. The two functions are step-like because they are computed at hourly intervals and are held constant throughout each hour (for the purposes of the simulation). The total
planned power starts from a value close to zero and takes 21 hours to first reach the predicted demand level. The reason is that generators begin from a cold start on the initial day. The initial value is not zero because there is still planned wind power during the first hour of the first day, and wind power does not begin from a cold start.

Figure 6-1: Planned Power vs. Predicted Demand (5.2%)

The total planned power is the solution planned by the day-ahead model and includes planned power from slow generators, planned power from fast generators, and predicted wind power. The planned power from the slow generators is implemented according to the day-ahead solution and becomes part of actual power. The planned fast power from the hour-ahead model, on the other hand, is rescheduled by the hour-ahead model. Similarly, the predicted wind power is eventually replaced by the actual wind power simulated within each hour.

Even without the readjustment of the planned power, however, it still does not completely satisfy the predicted demand. Figure 6-2 is the previous graph zoomed in on
hours 323 through 330 (day 13) of the simulation. There is not enough total planned power to satisfy predicted demand during this interval. If implemented, the shortage of the day-ahead solution as measured in sub-hourly increments would total approximately 4 million MW. Combined with the fact that the actual demand and wind differ from their predicted values, it becomes clear that, despite serving as a good approximation, the day-ahead solution is not an adequate implementable schedule.

If the planned power is not updated and is implemented to satisfy actual demand, the total simulation shortage is about 18 million MW. This constitutes a 350% increase compared to the day-ahead schedule applied to predicted demand. Figure 6-3 shows that the day-ahead schedule often misses the peak values of the actual demand. These shortages cause significant penalties and increase the total cost of simulation. The day-ahead schedule, therefore, should be modified on a smaller time scale to reduce the difference between planned power and actual demand.

Figure 6-2: Planned Power vs. Predicted Demand (Zoomed, 5.2%)
Taking into account actual wind values and the implemented day-ahead slow generation, the hour-ahead model adjusts the fast generator output in sub-hourly increments. Figure 6-4 shows the result, in which total shortage is reduced over 90% to about 328000 MW.
Figure 6-5 is a graph of actual power versus actual demand zoomed in on hours 323 to 330 (day 13) of the simulation. The actual power almost exactly matches the actual demand, differing in only three increments on this interval (two of these differences are easily visible in the graph). In the entire 20 day simulation, in fact, the actual power exactly matches the actual demand in 4630 out of the 5760 total five minute increments, or approximately 80% of the time.

![Actual Power vs. Actual Demand](image)

**Figure 6-5: Actual Power vs. Actual Demand (Zoomed, 5.2%)**

### 6.1.2 Explanation of Shortages

In the 5.2% simulation, the time increments during which actual power does not exactly match actual demand are often the first increment of the hour. Not including day 0 of the simulation, 177 shortages occur at time increment 0, totaling about 223000 MW. In contrast, only 87 potential shortages occur on all other time increments 1 through 11 inclusive, totaling about 105000 MW: less than half of the former amount. This observation is explained by a limitation of this hour-ahead model implementation.
Consider the actual power output during the last 30 minutes of hour 430 and the first 30 minutes of hour 431 (day 17 of the simulation) in Figure 6-6. Time increment 0 corresponds to the first five minutes of hour 431, which incurs a shortage of 3989 MW shown on the primary axis. The shortage occurs as a result of the horizon of the hour-ahead model.

Figure 6-6: Explanation of Shortage during Hour 431

The hour-ahead model solves a mixed integer linear program with a one hour horizon, meaning that all 12 of its solutions – one per five minute increment – are implemented. In this implementation, the hour-ahead model cannot foresee the planned slow generation of the next hour, nor could the day-ahead model have foreseen the ending fast generation of the current hour when generating the day-ahead schedule. If the difference in planned slow generation between this hour and the next hour is not great, or if the difference between actual fast generation between this hour and the planned fast generation of the next hour is not great, then no major problems arise. If, on the other hand, a significant discrepancy in the aforementioned values exists, shortages may occur.
Hour 431 of the simulation, illustrated in the above figure, is an example of the latter case. The planned slow power decreases by a significant amount between hours 430 and 431, but the day-ahead schedule had anticipated that fast power (“planned fast power”) was at a higher value than it actually was (“fast power”) at the start of hour 431, which would have cushioned the significant decrease in planned slow power. The hour-ahead model, however, is able to satisfy the actual demand with lower generation and, as a result, decreases the fast power from time increment 6 through 11 (in this implementation, the hour-ahead model cannot change the amount of slow power). When hour 431 arrives, there is not enough power to satisfy actual demand because both the slow power and fast power are too low, hence the shortage at time increment 0. It takes one time increment for the hour-ahead model to incorporate this information and ramp up fast power accordingly, and the shortage disappears by time increment 1.

The difference between actual and predicted wind is an indirect cause of this phenomenon: large predicted wind power causes the day-ahead model to schedule low amounts of slow power. If actual wind power is much smaller during that interval, the fast power required to satisfy actual demand is much larger, leading to shortages.

A potential fix for this limitation is increasing the horizon of the hour-ahead model by five minutes but not implementing the solution of the last time increment. Another potential method is to allow the hour-ahead model to modify the planned slow generation by ramping up or down the slow generators that are already online.

6.1.3 Distribution of Generated Power

Figure 6-7 highlights the difference in magnitude between the different sources of power. The actual fast power tends to exceed the planned fast power, but each is on
average several multiples lower than the slow power. This result makes sense because slow power is used to satisfy the baseload, which accounts for the majority of the power supplied. Fast power, on the other hand, is used to satisfy the peakload, which is more volatile but smaller in magnitude.

![Slow Power vs. Fast Power](image)

**Figure 6-7: Slow Power vs. Fast Power (5.2%)**

### 6.1.4 Percentage Difference in Overages and Shortages

Figure 6-8 displays the percentage difference between actual power and actual demand. Negative values indicate that curtailments in actual demand are necessary to prevent shortages. The majority of the largest shortages occur in day 0, when the generators are getting up to speed. Otherwise, there are periodic overages and shortages throughout the simulation.
Percentage difference omitting day 0 of the simulation is graphed with actual wind in Figure 6-9. With the exception of day 7, most of the shortages coincide with low amounts of actual wind.
6.1.5 Average and Instantaneous Generator Status

Figure 6-10 displays the number of fast generators that are warming up, online, and off at any given time. An average 13 generators are warming up at any given time, which is much lower than the average number at any given time that are online or off (269 and 323, respectively). Note that the number of online generators cycles rapidly between high and low values, reflecting their ability to satisfy peakload due to their short minimum warm-up times. Despite the fact that most of the fast generators have minimum warm-up and online times shorter than one hour, the large quantity of fast generators seems to eliminate the need of rapid cycling on an hourly scale.

![Number of Generators by Status](image)

**Figure 6-10: Distribution of Generator Status (5.2%)**

It is helpful to examine a typical generator’s continuous state throughout the entire simulation and draw conclusions about how it operates. Figure 6-11 depicts the status of generator 28 for each sub-hourly time increment of the simulation. On the y-
axis, 1 indicates that the generator is online, 0.5 indicates that the generator is warming up, and 0 indicates that the generator is off.

The generator immediately goes to the warming up state at the beginning of the simulation and goes online and turns off according to the hour-ahead schedule. Although the generator spends the majority of its time online, it does rapidly cycle between the off, warming up, and online states many times during the simulation. This characteristic may be due to the fact that this implementation of the hour-ahead model does not factor in warm-up costs, so generators are not penalized for frequently turning off and starting back on. Note also that the generator does not spend long consecutive time increments in the off state, suggesting that this generator is used frequently in the simulation.

Figure 6-11: Status of Generator 28 (5.2%)
6.2 5.3% Wind with Brownian Bridge Simulation

The second simulation is run for 20 days with 5.3% wind penetration and linear interpolation of actual demand values. The sub-hourly actual wind values, however, are simulated with a Brownian bridge. The purpose of this test is to simulate the volatile characteristic of actual wind values more accurately and assess its effect on the simulation. Linearly interpolating between hourly wind values makes the intra-hour values much less volatile than they are in reality. Although the hour-ahead model nonetheless treats these sub-hourly values as true values, the resulting volatility poses a challenge for the hour-ahead model because the demand that must be satisfied by fast generators becomes more volatile.

Introducing the volatility through the sub-hourly wind values is more realistic than introducing it through the sub-hourly demand values because demand forecasts, in practice, are much more accurate than wind forecasts. It is not necessary, furthermore, to introduce noise in both components because this test is merely a heuristic for observing what would happen as a result of increased volatility. In this simulation, therefore, the sub-hourly actual demand values are still generated using linear interpolation.

6.2.1 Methodology

Let \( \sigma^2 \) be the hourly variance used for simulating the sub-hourly actual wind values. Let \( Z_1 \ldots Z_{T-1} \) be \( i.i.d \) random variables drawn from a \( N(0,1) \) distribution: each is a standard normal variable with mean 0 and variance 1. Then \( \tilde{p}_{d,h}^W \) is a \( T \times 1 \) simulated Brownian motion with the first element fixed at \( W^A(f(d, h)) \).
Define $d\tilde{p}$ as the initial adjustment to pin the upper endpoint.

$$d\tilde{p} = \frac{W^A(f(d, h + 1)) - \tilde{p}_{d,h}(T - 1)}{T}$$

N.B.: During the last hour of the last day of simulation, a different value is substituted for $W^A(f(d, h + 1))$ since $W^A$ has size $D^s \times T^d$.

Let $\tilde{p}_{d,h}$ be the vector modified by the initial adjustment. Then $\tilde{p}_{d,h}$ is an effective simulated Brownian motion fixed between $W^A(f(d, h))$ and $W^A(f(d, h + 1))$, except the latter value does not appear as the last element of this wind vector but rather as the first element of the wind vector for the next hour. This convention is used to avoid repeating $W^A(f(d, h + 1))$.

$$\tilde{p}_{d,h}(s) = \begin{cases} W^A(f(d, h)) & \text{if } s = 0 \\ \tilde{p}_{d,h}(s - 1) + \frac{\sigma}{\sqrt{T}}Z_s & \text{otherwise} \end{cases}, \forall s \in [0, T - 1]$$

Finally, wind power cannot be negative. Define the secondary adjustment $d\tilde{p}$ as the minimum of all negative values in $\tilde{p}_{d,h}$.

$$d\tilde{p} = \begin{cases} 0 & \text{if } \tilde{s}_{d,h} = \emptyset \\ \min_s \tilde{s}_{d,h} & \text{otherwise} \end{cases}$$

The secondary adjustment is zero if $\tilde{p}_{d,h}$ has no negative values. All of the simulated values are adjusted so that the minimum is not negative. Recall that $p_{d,h}$ is the vector of simulated actual wind values, which is an input to the hour-ahead model.

$$p_{d,h}(s) = \begin{cases} \tilde{p}_{d,h}(s) & \text{if } s = 0 \\ \tilde{p}_{d,h}(s) - d\tilde{p} & \text{otherwise} \end{cases}, \forall s \in [0, T - 1]$$
This method of generating $p_{d,h}^W$ is different from simulating a Brownian bridge for several reasons, primarily that $p_{d,h}^W$ is not pinned on both endpoints of the same vector and that simulated values are adjusted so that the smallest value is zero. Note that the latter is not unreasonable in the context of this simulation because actual wind power can be zero. Nevertheless, this method is referred to as Brownian bridge simulation for convenience.

The variance used for this simulation is the average of the hourly wind variance for each day over all days in the simulation. Let $\sigma_0^2 \ldots \sigma_{D-1}^2$ be the sample variance of the members in the sets $\{W^A(0, h)\}_{h=0}^{T-1} \ldots \{W^A(D - 1, h)\}_{h=0}^{T-1}$, respectively. In other words, $\{W^A(k, h)\}_{h=0}^{T-1}$ is the set of $T = 24$ wind values for day $k$, and $\sigma_k^2$ is the sample variance of that set. Then the variance used for Brownian bridge simulation is:

$$\sigma^2 = \frac{1}{D} \sum_{k=0}^{D-1} \sigma_k^2$$

Although a $x$-day rolling average of hourly variance would more closely capture the correlated nature of wind, this approach is preferred for its simplicity. More sophisticated algorithms for intra-hourly wind simulation are not necessary because the main purpose of this subsection is to assess the effect of larger wind volatility on simulation performance.

### 6.2.2 Results

The actual wind power from the Brownian bridge simulation is much more volatile than the original wind power, as shown in Figure 6-12. Although the average wind power per time increment increases 1.2%, the variance increases 14.3%. The
original wind penetration is 5.2%, whereas the new wind penetration increases to 5.3% due to a larger proportion of wind power in the system. For 32 time increments in the simulation, the actual wind power is zero; this number comprises 0.6% of the total time increments in the simulation.

Figure 6-12: Actual Wind with Brownian Bridge (5.3%)

The cost of the 5.3% Brownian bridge simulation is $4.19 \times 10^{10}$, which is 5.8% more expensive than the 5.2% base case simulation. The increase in cost is expected because volatile wind power makes it more difficult for the hour-ahead model to prevent shortages. Figure 6-13 displays the percentage difference between total actual power and total actual demand for the Brownian bridge simulation and the original simulation. Although many time increments in the Brownian bridge case experience shortages where none existed in the base case, there are also time increments where the base case has shortages but the Brownian bridge case does not.
Thus, a temporal pattern between original and new shortages is not obvious. The Brownian bridge simulation does, however, increase the number of shortages and overages, which is shown in Table 6-1. The total shortage and overage also increase, but the average shortage and overage decrease. These results suggest that when wind is volatile, it is more difficult for the hour-ahead model to commit power that exactly matches demand, leading to more frequent — yet smaller — shortages and overages. Indeed, the number of time increments where actual power exactly matches actual demand decreases from 4518 to 4246 when Brownian bridge simulation is used. Note also that the percentage increase in total shortage is over six times as large as the percentage increase in total overage. The increased volatility of wind, therefore, affects shortages more than overages, which increases cost.
Table 6-1: Shortages and Overages with Brownian Bridge (5.3%)

<table>
<thead>
<tr>
<th></th>
<th>5.3% Wind (Brownian Bridge)</th>
<th>5.2% Wind (Original)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>468</td>
<td>264</td>
<td>7.7%</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>532000</td>
<td>328000</td>
<td>6.2%</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>1140</td>
<td>1240</td>
<td>-8.5%</td>
</tr>
<tr>
<td># overages</td>
<td>758</td>
<td>690</td>
<td>9.9%</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>164000</td>
<td>162000</td>
<td>1.0%</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>216</td>
<td>235</td>
<td>-8.0%</td>
</tr>
</tbody>
</table>

Table 6-2 confirms that the effect of shortages outweighs the effect of overages. The increase in shortage penalty is 62%, which is over 50 and 100 times greater than the increase in the cost of slow and fast generation, respectively.

Table 6-2: Cost Distribution with Brownian Bridge (5.3%)

<table>
<thead>
<tr>
<th>Cost ($10^8) (excluding day 0)</th>
<th>5.3% Wind (Brownian Bridge)</th>
<th>5.2% Wind (Original)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>239</td>
<td>238</td>
<td>0.6%</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>126</td>
<td>125</td>
<td>1.1%</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>53</td>
<td>33</td>
<td>62%</td>
</tr>
<tr>
<td>Total cost</td>
<td>419</td>
<td>396</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

Table 6-3 shows the changes in generation due to Brownian bridge simulation. With large wind volatility, the emphasis shifts toward satisfying peakload, which leads to an increase in total fast generation at the expense of total slow generation. Changes in committed generation occur in 1000+ time increments for both slow and fast generation. Volatility in actual wind power is an important part of how the hour-ahead schedule is created, irrespective of whether actual or predicted values are used as inputs to the model.
Table 6-3: Changes in Generation with Brownian Bridge (5.3%)

<table>
<thead>
<tr>
<th>Changes in Generation (excluding day 0)</th>
<th>5.3% Wind (Brownian Bridge)</th>
</tr>
</thead>
<tbody>
<tr>
<td>% change in total slow generation (10^6 MW)</td>
<td>-0.26%</td>
</tr>
<tr>
<td># time increments with increased slow generation</td>
<td>1668</td>
</tr>
<tr>
<td># time increments with decreased slow generation</td>
<td>3084</td>
</tr>
<tr>
<td>% change in total fast generation (10^6 MW)</td>
<td>0.55%</td>
</tr>
<tr>
<td># time increments with increased fast generation</td>
<td>2884</td>
</tr>
<tr>
<td># time increments with decreased fast generation</td>
<td>2556</td>
</tr>
</tbody>
</table>

6.3 20.4% Wind

The next simulation is run for 20 days with 20.4% wind penetration. Linear interpolation is used to generate sub-hourly wind and demand values. Since obtaining accurate five minute predictions of wind and demand is beyond the scope of this thesis, the use of linear interpolation suffices for the rest of these simulations.

The total cost of the 20.4% simulation is $2.91 \times 10^{10}$, or about 26.4% cheaper than the 5.2% simulation. Wind power is assumed to be free, so 20.4% of the total actual power is supplied at no cost, compared to only 5.2% of the previous total power. At the same time, however, wind penetration has not yet increased to the point where volatility causes shortages that significantly increase total cost. Figure 6-14 compares the actual wind for the two simulations.
Figure 6-15 contrasts actual slow and actual fast power in the 20.4% and 5.2% simulations. Both types of 20.4% power are in general smaller in magnitude because less fast and slow power is needed. The 20.4% slow power, in addition, is more volatile. The baseload fluctuates more because 20.4% wind is more volatile than 5.2% wind. Furthermore, the amount of 20.4% fast power is almost zero for the time increments where 20.4% wind and slow power are sufficient to satisfy actual demand.
The percentage difference between actual power and actual demand again appears to be highly correlated with the amount of available wind power, as seen in Figure 6-16. Amounts of wind power over 16000 MW lead to overages that tend to last multiple time increments; lower levels of wind tend to lead to brief shortages. Overages occur when the wind and slow power exceed the actual demand. During these time increments, fast power cannot be ramped down further (otherwise, the hour-ahead model would incur larger costs and would not have found the optimal solution). Slow power, on the other hand, cannot be modified by the hour-ahead model. Total generation costs may be reduced by allowing the hour-ahead model to modify slow generation.

Figure 6-16: Percentage Difference vs. Actual Wind (20.4%)

Figure 6-17 compares the percentage differences between the 20.4% and 5.2% simulations. The former has more overages (2591 versus 690) and fewer shortages (158 versus 264) due to the increased amount of available wind power.
6.4 40.0% Simulation

The third simulation is run for 20 days with 40.0% wind penetration. The resulting cost is $4.02 \times 10^{10}$, which is larger than each of the other simulations. With 40.0% wind penetration, the magnitude of actual wind power is much larger, with a mean of 39779 MW compared to 18800 MW (20.4% simulation) and 4736 MW (5.2% simulation). Figure 6-18 shows that wind power is also more volatile at the larger wind penetration, with a standard deviation of 18951 MW compared to 8957 MW and 2256 MW, respectively. Total generation costs are the largest of the three wind penetration simulations because the wind volatility is now large enough that it causes problems for the hour-ahead model in committing fast generation, despite the cost reductions of using free wind.
The 40.0% fast power in Figure 6-19 is less than 100 MW almost twice as often: 2884 time increments compared to 1460 in the 20.4% simulation. 40.0% wind power and slow power, therefore, are sufficient to satisfy demand half of the time, since wind does not contribute a steady 40.0% to actual power but rather varying amounts.

Figure 6-19: Slow Power vs. Fast Power (40.0%, 5.2%)
The overages from the 40.0% simulation dwarf those from the 20.4% and 5.2% simulations, as seen in Figure 6-20. Although the overages are larger and more frequent, total generation costs still decrease to $1.84 \times 10^{10}$ due to higher wind penetration.

![Figure 6-20: Percentage Difference (40.0%, 20.4%, 5.2%)](image)

### 6.5 Comparison of Wind Penetration Simulations

The three levels of wind penetration tested are 5.2%, 20.4%, and 40.0%. In general, overages increase in frequency and magnitude as wind penetration increases, whereas shortages display a more complex relationship due to the dip in their frequency and magnitude at 20.4% wind. Generation costs decrease with wind penetration, whereas shortage penalties tend to increase with wind penetration (20.4% wind is again an exception). The average number of online generators decreases with wind penetration, but the average number of warming-up generators is approximately constant. These findings are summarized below in greater detail.
6.5.1 Shortages and Overages

Table 6-4 presents data on shortages and overages rounded to three significant digits for each of the three wind penetration simulations. Although the average shortage increases in size as wind penetration increases from 5.2% to 20.4%, shortages decrease in frequency and total size because a higher amount of wind is available in the system. As wind penetration increases to 40.0%, however, the hour-ahead model encounters more difficulty coping with the higher volatility of wind, which increases the frequency, total size, and average size of shortages.

The relationship between overages and wind penetration is more straightforward. As wind penetration increases, the frequency, total size, and average size of overages all increase. Furthermore, the marginal increase in total and average overage also increases. A drawback of this implementation of the hour-ahead model is that it cannot adjust slow power. Thus, when actual wind and slow power greatly exceed actual demand, overages occur and power is wasted.

Table 6-4: Shortages and Overages (5.2%, 20.4%, 40.0%)

<table>
<thead>
<tr>
<th>Total (excluding day 0)</th>
<th>5.2% Wind</th>
<th>20.4% Wind</th>
<th>40.0% Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>264</td>
<td>158</td>
<td>445</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>328000</td>
<td>247000</td>
<td>2190000</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>1240</td>
<td>1560</td>
<td>4920</td>
</tr>
<tr>
<td># overages</td>
<td>690</td>
<td>2591</td>
<td>3376</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>162000</td>
<td>893000</td>
<td>5110000</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>235</td>
<td>3450</td>
<td>15100</td>
</tr>
</tbody>
</table>
6.5.2 Cost

Table 6-5 presents cost distribution data rounded to three significant digits for the three simulations. The total cost for the 20.4% simulation is less than that of the 5.2% simulation due to greater wind penetration and the assumption that wind power costs nothing. Slow power cost and fast power cost are lower because more wind is used to satisfy actual demand. Shortage penalties also decrease due to fewer shortages. Costs increase, however, in the 40.0% simulation. Although slow power cost and fast power cost decrease further compared to the 20.4% simulation (due to double the wind penetration), an increased frequency and severity of shortages causes the shortage penalty to be more than six times as large as that of the other simulations.

It is worth noting, however, that the total cost is approximately the same for the 5.2% and 40.0% simulations (the cost of the latter represents only a 1.5% increase over the cost of the former). Rather, only the breakdown of the costs is different: at the lower wind penetration, almost 92% of the cost comes from generation, whereas at the higher wind penetration, less than 46% is due to generation, with the remainder resulting from shortage penalties.

Table 6-5: Cost Distribution (5.2%, 20.4%, 40.0%)

<table>
<thead>
<tr>
<th>Cost ($10^k) (excluding day 0)</th>
<th>5.2% Wind</th>
<th>20.4% Wind</th>
<th>40.0% Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>238</td>
<td>195</td>
<td>116</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>125</td>
<td>71</td>
<td>68</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>33</td>
<td>25</td>
<td>219</td>
</tr>
<tr>
<td>Total cost</td>
<td>396</td>
<td>291</td>
<td>402</td>
</tr>
</tbody>
</table>
6.5.3 Generator Status

Table 6-6 depicts the average number of fast generators in each state during each simulation. As explained earlier, the average number of online generators decreases as the wind penetration increases. The larger decrease comes from the initial increase in wind penetration, suggesting that the marginal difference between implementing 20.4% versus 40.0% wind penetration is not as large as the initial hurdle of establishing 20.4% wind penetration. Correspondingly, the average number of off generators increases with wind penetration. The average number of warming-up generators, however, is roughly constant with wind penetration, suggesting that the hour-ahead model maintains a safety queue of stable size when scheduling generators.

Table 6-6: Generator Status (5.2%, 20.4%, 40.0%)

<table>
<thead>
<tr>
<th>Average Number of Generators</th>
<th>5.2% Wind</th>
<th>20.4% Wind</th>
<th>40.0% Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warming up</td>
<td>13</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>Online</td>
<td>269</td>
<td>159</td>
<td>132</td>
</tr>
<tr>
<td>Off</td>
<td>323</td>
<td>433</td>
<td>464</td>
</tr>
</tbody>
</table>
7 Designing and Testing a Horizon-Increasing Heuristic

The hour-ahead model’s tendency to cause shortages at the beginning of the hour, which was analyzed in the 5.2% simulation, also exists in other wind penetration simulations. This chapter explains the limitation of the hour-ahead model’s horizon, proposes a heuristic to amend the problem, and analyzes its effectiveness. In addition, a method to increase the horizon without using a heuristic is proposed, and corresponding changes to the simulation model are suggested.

7.1 Revisiting the Horizon Problem

Figure 7-1 depicts the power distribution during the last 30 minutes of hour 238 and the first 30 minutes of hour 239 (day 9) of the 40.0% simulation. The blue bar is measured on the primary y-axis and represents the difference between actual power and actual demand. Negative values represent shortages. Fast generation is measured on the primary y-axis; slow power is measured on the secondary y-axis. The hour-ahead model for hour 238 does not anticipate that significantly less slow power is available for hour 239, which is scheduled by the day-ahead model. As a result, the hour-ahead model does not increase fast generation during the last time increments of hour 238: in fact, slight overages already occur during that time, so there is no need to ramp up generation and
incur unnecessary costs. When hour 239 arrives, however, the combined actual slow and fast power is too low, and a shortage occurs. The hour-ahead model increases fast power significantly for the next two time increments, but the system experiences an additional shortage before demand is cleared.

Figure 7-1: Explanation of Shortage (40.0%)

The gap between fast power and planned fast power results from the large difference in actual and predicted wind. The day-ahead model plans fast power assuming 28593 MW of wind power, but the actual wind power is only 17153 MW, or 40% less than the predicted value. As a result, the actual fast power required to clear demand is much larger than the planned value. These discrepancies are more common in the 40.0% simulation than the 5.2% simulation because more wind power is used despite the inaccuracies of wind predictions.

If the hour-ahead model were given the deterministic information that the day-ahead model had scheduled less slow power for the next hour, then the hour-ahead model
could commit excess fast power for the last increment of the current hour. Then the shortage at time increment 0 of the next hour would not be as large. In other words, increasing the horizon of the hour-ahead model would alleviate the problem of shortages. Although doing so would incur larger generation costs for that hour due to overages, the total simulation cost would decrease because shortages would be avoided during the next hour. The tradeoff of more overage in the current hour for less shortage in the next hour is beneficial because the cost of shortage is much larger than the cost of overage. It is hypothesized, therefore, that a heuristic that increases the horizon of the hour-ahead model would decrease the total cost of simulation.

7.2 Designing a Heuristic to Increase the Horizon

The horizon of this implementation of the hour-ahead model is one hour separated into $T = 12$ five minute increments. The solution to the mixed integer linear programming formulation of the hour-ahead model is given in 12 parts, and each part is implemented in the simulation. A heuristic that increases the horizon of the problem yet is compatible with the algorithm, therefore, must keep the same number of solutions and implement all of them. The heuristic that is proposed below artificially increases the horizon of the problem by increasing the demand to be satisfied during the last time increment if and only if planned slow power decreases in the next hour.

From the simulation model, recall that $p^S_{d,h}$ is the total slow generation in MW for hour $h$ of day $d$, which is scheduled by the day-ahead model of day $d$ and implemented by the hour-ahead model of cumulative hour $f(d, h)$. Also recall that $\lambda^H_{d,h}$ is the $T \times 1$ vector of sub-hourly demands in MW that must be satisfied by the hour-ahead model of
cumulative hour $f(d, h)$. Furthermore, recall that $\mathbf{x}_{d,h}^H = X^H, \pi(S_{d,h}^H, \lambda_{d,h}^H)$ is the solution of the hour-ahead model at cumulative hour $f(d, h)$.

Let $H_{d,h}^H$ be the extra generation to be satisfied by the hour-ahead model of hour $f(d, h)$, which is implemented as a heuristic to increase the horizon of the hour-ahead model. Let $\lambda_{d,h}^H$ be the adjusted sub-hourly demand vector after incorporating the heuristic. Let $\tilde{\mathbf{x}}_{d,h}^H$ be the adjusted solution of the hour-ahead model after incorporating the heuristic, and let $\tilde{S}_{d,h}^H$ be the adjusted state variable obtained from information that also incorporated the heuristic. Then the heuristic is implemented through the following method:

$$H_{d,h}^H = \begin{cases} p_{d,h}^S - p_{d,h+1}^S & \text{if } p_{d,h}^S - p_{d,h+1}^S > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall d \in [0, D^S - 1], h \neq T^d - 1$$

$$\lambda_{d,h}^H(s) = \begin{cases} \lambda_{d,h}^H(s) + H_{d,h}^H & \text{if } s = T - 1 \\ \lambda_{d,h}^H(s) & \text{otherwise} \end{cases} \quad \forall d \in [0, D^S - 1], h \neq T^d - 1, s \in [0, T - 1]$$

$$\tilde{\mathbf{x}}_{d,h}^H = X^H, \pi(\tilde{S}_{d,h}^H, \tilde{\lambda}_{d,h}^H)$$

This heuristic satisfies the requirement of compatibility with the functional form of the hour-ahead model: only the input changes. The extra generation $H_{d,h}^H$ is the difference between the current slow generation and the slow generation of the next hour, if this difference is positive. Adding this extra generation to the last sub-hourly component of the demand vector that is passed to the hour-ahead model forces the hour-ahead model to commit this extra generation whenever possible, which in turn reduces the shortage in the first time increment of the next hour if it exists. If this difference is negative, extra generation is likely not needed because slow generation is increased in the next hour. The heuristic is only implemented for hours $h \in [0, 22]$ because during hour 23 of each day, the day-ahead schedule for the next day has not yet been created, so the
slow generation of the next hour is therefore undetermined. In order to avoid looking into the future, no heuristic is implemented during hour 23.

This heuristic does not guarantee the reduction of shortages. A shortage may not exist in the first increment of the next hour despite less slow generation, because actual wind power during the next hour may be large. Conversely, a shortage may occur in the next hour even if slow generation is increased because actual wind power is small; this case is not covered by the heuristic. It is also possible that increasing the fast power during the last hour of each day would induce the day-ahead model to schedule even less slow power for the next day.

Nevertheless, the heuristic may reduce costs. The main idea of the heuristic is that the cost of shortage is larger than the cost of overage, thus total cost may be reduced by forcing more overage to reduce shortage. The asymmetric nature of the costs makes the hour-ahead model similar to the newsvendor problem. Simulations that incorporate the heuristic are analyzed below.

### 7.3 5.2% Simulation with Heuristic

When the 5.2% simulation is run with the horizon-increasing heuristic, the total cost decreases from $3.96 \times 10^{10}$ to $3.89 \times 10^{10}$, or a reduction of 1.67%. The heuristic leads to significant changes in the division of generated power, shortage and overage statistics, and cost distribution.

The amount of generation changes because both the day-ahead and hour-ahead models operate differently with the heuristic. Figure 7-2 shows the amount of slow power generated throughout the simulation for both the heuristic and the original
simulations. A positive percentage change between the two values indicates that more slow power is generated with the heuristic. Although the two sets of data are identical for the majority of the simulation, they differ during the last five days. Less slow power is generated overall (about 81600 fewer MW, or 0.02% less slow generation than the base case). This decrease in slow generation could be due to the increase in fast power, which diminishes the need for the day-ahead model to schedule as much slow power.

![Image of the chart showing the effect of heuristic on slow power (5.2% Wind)](image)

**Figure 7-2: Slow Power with Heuristic (5.2%)**

The changes to fast power are more pronounced, as shown in Figure 7-3. Variations occur throughout the entire simulation, and the percentage change is often one or two orders of magnitude larger than in the case of slow power. The reason for the large percentage changes is that the heuristic directly affects the fast power that is committed by the hour-ahead model. In addition, the magnitude of original fast output is lower than that of original slow output, so any changes would constitute a larger
percentage increase, *ceteris paribus*. Increases in fast output – the positive values – are much more common than decreases.

N.B.: An outlier of 280% at the last increment of cumulative hour 288 (day 12) has been removed from the graph to maintain the secondary axis scaling.

The larger percent increases in fast generation tend to occur when the original fast generation is relatively low. The reason is that the hour-ahead model with the heuristic is more generous in committing fast generation – especially when the originally committed fast generation is low – in order to prevent potential shortages.

The shortage and overage statistics change as a result of the previously described changes to power generation. Figure 7-4 compares the percentage difference between total actual power and total actual demand in the 5.2% simulation with and without the heuristic. Negative values represent shortages. The frequency of shortages decreases when using the heuristic; the frequency of overages, on the other hand, increases.
The percentage differences are analyzed in further detail in Table 7-1, which displays the shortage and overage data with and without the heuristic. The heuristic achieves the goal of reducing shortages (frequency, total size, and average size), at the expense of increasing overages. The percentage changes in overages are larger in magnitude than their corresponding changes in shortages.

**Table 7-1: Shortages and Overages with Heuristic (5.2%)**

<table>
<thead>
<tr>
<th>Total (excluding day 0)</th>
<th>5.2% Wind (Original)</th>
<th>5.2% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>264</td>
<td>244</td>
<td>-7.6%</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>328000</td>
<td>255000</td>
<td>-22%</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>1240</td>
<td>1050</td>
<td>-16%</td>
</tr>
<tr>
<td># overages</td>
<td>690</td>
<td>865</td>
<td>25%</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>162000</td>
<td>337000</td>
<td>108%</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>234</td>
<td>390</td>
<td>66%</td>
</tr>
</tbody>
</table>
Table 7-2 displays the rounded values of the cost distribution for the 5.2% simulation with and without the heuristic. The total cost of slow generation increases by only 0.3%, and the total cost of fast generation increases slightly more due to more output. The total shortage penalty decreases significantly and outweighs the increase in generation costs, which brings down the total cost and reduces the proportion of total cost attributed to shortages.

Table 7-2: Cost Distribution with Heuristic (5.2%)

<table>
<thead>
<tr>
<th>Cost ($10^8) (excluding day 0)</th>
<th>5.2% Wind (Original)</th>
<th>5.2% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>238</td>
<td>238</td>
<td>0.3%</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>125</td>
<td>126</td>
<td>0.8%</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>33</td>
<td>25</td>
<td>-24%</td>
</tr>
<tr>
<td>Total cost</td>
<td>396</td>
<td>389</td>
<td>-1.7%</td>
</tr>
</tbody>
</table>

7.4 20.4% Simulation with Heuristic

When the 20.4% simulation is run with the horizon-increasing heuristic, the total cost decreases from $3.91 \times 10^{10}$ to $3.84 \times 10^{10}$, or a reduction of 2.26% compared to the original 20.4% simulation. The heuristic does not change the scheduled slow power.

According to Figure 7-5, it is not the case that the 20.4% heuristic simulation fails to increase fast power, which may have otherwise induced the day-ahead model to schedule less slow power. The heuristic does cause the hour-ahead model to commit more fast generation. The 20.4% heuristic simulation, in fact, commits more additional fast generation relative to the 20.4% base case than the 5.2% heuristic does relative to the 5.2% base case, both in absolute and percentage terms: an additional 372400 MW
compared to 329700 MW, or equivalently, an additional 0.69% of base case fast generation compared to 0.36%.

The largest negative percentage change in fast generation is \(-97\%\), which occurs during hour 54 (day 2) of the simulation. The negative percentage changes are not visible from the graph because the positive percentage changes dwarf the negative changes. The former reach levels as high as 13350% of base case fast generation. The negative percentage changes, in contrast, are capped at \(-100\%\) since generation cannot be negative. Unlike the 5.2% simulation, outliers in the 20.4% simulation cannot be removed because they account for a much larger proportion of the increases in fast generation: 224 of the 600 positive percentage increases exceed 200% in this heuristic simulation, compared to 1 out of 1029 in the 5.2% heuristic simulation.

The base case fast generation in the 20.4% simulation is almost zero for many time increments because wind and slow generation are enough to satisfy demand. An
increase in fast generation from these low values causes the percentage increase to often exceed 1000%. The outliers occur, therefore, because the hour-ahead model commits power more liberally to anticipate shortages, which achieves the heuristic’s objective.

Figure 7-6 compares the percentage difference between total actual power and total actual demand for the 20.4% simulation with and without the heuristic. Negative values represent shortages. With the heuristic, overages are more common and larger in magnitude. Shortages, on the other hand, are less severe.

Table 7-3 shows the rounded statistics for shortages and overages in the 20.4% simulation. Although the number of shortages increases with the heuristic, the total and average shortage both decrease greatly. In addition, the number of overages and total overage increase, which is expected, but the average overage decreases. The increase in number of shortages and the decrease in average overage are the different outcomes compared to the 5.2% heuristic simulation results.
Table 7-3: Shortages and Overages with Heuristic (20.4%)

<table>
<thead>
<tr>
<th>Total (excluding day 0)</th>
<th>20.4% Wind (Original)</th>
<th>20.4% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>158</td>
<td>165</td>
<td>4.4%</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>247000</td>
<td>175000</td>
<td>-29%</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>1560</td>
<td>1060</td>
<td>-32%</td>
</tr>
<tr>
<td># overages</td>
<td>2591</td>
<td>2759</td>
<td>6.5%</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>8930000</td>
<td>9230000</td>
<td>3.4%</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>3450</td>
<td>3350</td>
<td>-2.9%</td>
</tr>
</tbody>
</table>

Table 7-4 shows the rounded cost distribution of the 20.4% simulation. The total cost of slow power does not change, whereas the total cost of fast power increases slightly and the total shortage penalty decreases greatly.

Table 7-4: Cost Distribution with Heuristic (20.4%)

<table>
<thead>
<tr>
<th>Cost ($10^{8}$) (excluding day 0)</th>
<th>20.4% Wind (Original)</th>
<th>20.4% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>195</td>
<td>195</td>
<td>0%</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>71</td>
<td>72</td>
<td>0.8%</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>24.7</td>
<td>17.5</td>
<td>-29%</td>
</tr>
<tr>
<td>Total cost</td>
<td>291</td>
<td>284</td>
<td>-2.3%</td>
</tr>
</tbody>
</table>

7.5 40% Simulation with Heuristic

The 40% simulation consists of 40.0% wind penetration for the base case and 39.9% wind penetration for the heuristic case. The wind penetration decreases because the heuristic generates enough additional fast power such that total wind power becomes
a smaller percentage of total power. For convenience, these simulations are referred to as the 40% simulation with and without the heuristic.

The total cost decreases from $4.02 \times 10^{10}$ to $3.89 \times 10^{10}$ after implementing the heuristic, or a reduction of 3.29%. As with the 20.4% simulation, the slow generation with the heuristic is identical to the original slow generation.

Figure 7-7 shows the change in fast generation as a result of the heuristic for the 40% simulation. A positive percent change indicates that the hour-ahead model commits more fast generation with the heuristic. The large positive percent changes tend to occur when the heuristic fast power does not coincide with the original fast power; in the graph, the green bars tend to occur at the same time increments where the blue lines do not coincide with the red lines. In other words, the heuristic causes the hour-ahead model to commit fast generation where it previously does not commit generation, which is why the percent changes are very large—sometimes exceeding 10000%. This phenomenon results from the generosity with which the heuristic hour-ahead model commits fast generation to prevent possible shortages.

The largest negative percentage change in fast generation is $-90\%$ and occurs during hour 56 (day 2) of the simulation. As is the case with the 20.4% heuristic simulation, the negative percentage changes are not visible in the graph because they are dwarfed by the positive outliers. Out of 591 total time increments where committed fast power increases when using the 40% heuristic, 331 of those occurrences represent a percentage change exceeding 200%. Positive outliers become more common when wind penetration increases.
Figure 7-8 compares the percentage difference for the 40% simulation with and without the heuristic. Negative values indicate shortages. The number of shortages increases with the heuristic, but the total shortage decreases.
Table 7-5 presents the rounded shortage and overage data for the 40% simulation. As is the case with the 20.4% simulation, the number of shortages increases with the heuristic, but total and average shortage both decrease. The number of overages and total overage increase with the heuristic, but the average overage decreases.

Table 7-5: Shortages and Overages with Heuristic (40%)

<table>
<thead>
<tr>
<th>Total (excluding day 0)</th>
<th>40.0% Wind (Original)</th>
<th>39.9% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>445</td>
<td>461</td>
<td>3.6%</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>2190000</td>
<td>2110000</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>4920</td>
<td>4580</td>
<td>-6.8%</td>
</tr>
<tr>
<td># overages</td>
<td>3376</td>
<td>3472</td>
<td>2.8%</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>51100000</td>
<td>51600000</td>
<td>0.9%</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>15100</td>
<td>14900</td>
<td>-1.9%</td>
</tr>
</tbody>
</table>

Table 7-6 depicts the rounded cost distribution for the 40% simulation. The cost of slow power is again identical, whereas the cost of fast power increases by 1.3%. The shortage penalty decreases by a larger absolute value in the 40% simulation than in the 5.2% and 20.4% simulations, but the percentage decrease is smaller.

Table 7-6: Cost Distribution with Heuristic (40%)

<table>
<thead>
<tr>
<th>Cost ($10^8$) (excluding day 0)</th>
<th>40.0% Wind (Original)</th>
<th>39.9% Wind (Heuristic)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>116</td>
<td>116</td>
<td>0%</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>68</td>
<td>69</td>
<td>1.3%</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>219</td>
<td>205</td>
<td>-6.5%</td>
</tr>
<tr>
<td>Total cost</td>
<td>402</td>
<td>389</td>
<td>-3.3%</td>
</tr>
</tbody>
</table>
7.6 Comparison of Heuristic Simulations

This section compares the percentage changes of various metrics as a result of the heuristic for all three simulations. Table 7-7 shows the comparison of shortage and overage data with respect to the base cases. The percentage decrease in total shortage increases with higher wind penetration for the first two simulations but then decreases dramatically for the 40% simulation. The reason is that the size of the original total shortage for the 40% simulation is so large (2190000 MW compared to 328000 MW for the 5.2% simulation) that even though the heuristic decreases the total shortage by roughly the same amount (76200 MW compared to 72900 MW for the 5.2% simulation), the percentage decrease is much smaller.

The table also shows that the percentage change in the number of overages, total overage, and average overage actually decrease with increasing wind penetration, which may not be obvious. Although the absolute values become larger with wind penetration, this result implies that the hour-ahead model does not go overboard with its tendency to err on the side of overages, thereby limiting the increase in excess generation.

Table 7-7: Shortages and Overages (5.2%, 20.4%, 40%)

<table>
<thead>
<tr>
<th>% Change compared to Base Case (excluding day 0)</th>
<th>5.2% heuristic</th>
<th>20.4% heuristic</th>
<th>40% heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>-7.6%</td>
<td>4.4%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>-22%</td>
<td>-29%</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>-16%</td>
<td>-32%</td>
<td>-6.8%</td>
</tr>
<tr>
<td># overages</td>
<td>25%</td>
<td>6.5%</td>
<td>2.8%</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>108%</td>
<td>3.4%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>66%</td>
<td>-2.9%</td>
<td>-1.9%</td>
</tr>
</tbody>
</table>
Table 7-8 compares the percentage change in cost distribution with respect to the base cases. The cost of slow power does not change for the 20.4% and 40% heuristic simulations. The percentage change in the cost of fast power, unexpectedly, does not increase greatly with wind penetration. This observation may be an indicator that the heuristic does not generate additional power excessively. The percentage change in the shortage penalty decreases dramatically in the 40% simulation, again due to its initial large value in the 40% base case. Finally, the percentage decrease in the total cost of simulation increases with wind penetration, which implies an increasing marginal benefit of the heuristic and suggests that increasing the horizon of the hour-ahead model may reduce costs even with large wind penetration.

Table 7-8: Cost Distribution with Heuristic (5.2%, 20.4%, 40%)

<table>
<thead>
<tr>
<th>% Change compared to Base Case (excluding day 0)</th>
<th>5.2% heuristic</th>
<th>20.4% heuristic</th>
<th>40% heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>0.3%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>0.8%</td>
<td>0.8%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>-24%</td>
<td>-29%</td>
<td>-6.5%</td>
</tr>
<tr>
<td>Total cost</td>
<td>-1.7%</td>
<td>-2.3%</td>
<td>-3.3%</td>
</tr>
</tbody>
</table>

Table 7-9 shows the changes in fast generation due to the heuristic. The percentage increase in fast generation becomes larger with wind penetration because base case fast generation is the lowest for high wind penetration. The number of time periods where fast generation increases, however, actually decreases with wind penetration, which means that the additional generation is concentrated into fewer time periods. This result is also reflected in the distribution of outliers. Percentage increases in fast generation greater than 200% are classified as outliers, and percentage increases greater
than 10000% are classified as super-outliers. As wind penetration increases, outliers and super-outliers account for a larger percentage of the increases in fast generation.

Table 7-9: Changes in Fast Generation with Heuristic (5.2%, 20.4%, 40%)

<table>
<thead>
<tr>
<th>Fast Generation compared to Base Case (excluding day 0)</th>
<th>5.2% heuristic</th>
<th>20.4% heuristic</th>
<th>40% heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>% increase in total fast generation (10^6 MW)</td>
<td>0.36%</td>
<td>0.69%</td>
<td>1.3%</td>
</tr>
<tr>
<td># time increments with increased fast generation</td>
<td>1029</td>
<td>600</td>
<td>591</td>
</tr>
<tr>
<td># (%) time increments with % change &gt; 200%</td>
<td>0 (0%)</td>
<td>224 (37.3%)</td>
<td>331 (56.0%)</td>
</tr>
<tr>
<td># (%) time increments with % change &gt; 10000%</td>
<td>0 (0%)</td>
<td>7 (1.2%)</td>
<td>20 (3.4%)</td>
</tr>
</tbody>
</table>

The number of time increments with increased fast generation is a relatively small percentage of the total number of time increments. For the 40% simulation, they account for \( \frac{591}{5760} = 10.3\% \) of all time increments. Thus, the hour-ahead model does not increase fast generation at a majority of time periods, which is the heuristic’s intention (otherwise, too much power would be wasted). The heuristic is designed such that the increases in fast generation would ideally occur in the last time increment of the hour, *ceteris paribus*, to alleviate potential shortages yet minimize excess generation. In practice, \( \frac{64}{591} = 10.8\% \) of all increases occurred during the last time increment \( s = 11 \) of any hour (as well as \( \frac{34}{331} = 10.3\% \) of all outlier increases). These values are only slightly larger than what they would be if they followed a uniform distribution of \( \frac{1}{12} = 8.3\% \) across all time periods, suggesting that the heuristic should be further fine-tuned so that more increases occur at the end of each hour.
7.7 Generalization using a Tunable Parameter

If the difference between current slow generation and slow generation for the next hour is positive, the heuristic adds this difference to the demand that must be satisfied by the hour-ahead model. Suppose, instead, that a multiple of the difference is a more optimal quantity to add in order to balance the costs of overage and shortage. Let $\rho > 0$ be a tunable parameter. Then the heuristic can be generalized as follows:

$$H_{d,h}^H(\rho) = \begin{cases} 
\rho \times (p_{d,h}^S - p_{d,h+1}^S) & \text{if } p_{d,h}^S - p_{d,h+1}^S > 0 \\
0 & \text{otherwise} 
\end{cases} \forall d \in [0, D^s - 1], h \neq T^d - 1$$

Different values of $\rho$ lead to different simulation outcomes, so an optimal $\rho$ may be found. The previous formulation of the heuristic is $H_{d,h}^H = H_{d,h}^H(1)$ for all three wind penetration simulations. Now the tunable parameter is varied: $\rho = 1$, $\rho = 2$, and $\rho = 3$ are each tested for the 40% simulation. It is not claimed that any of the tested tunable parameters is optimal; rather, the intention is to show how varying the tunable parameter reveals tradeoffs in the underlying problem.

The wind penetration for each of the three tunable parameter simulations is 39.9%, but they are collectively referred to as the 40% simulation for convenience. The total costs are $\$3.892 \times 10^{10}$, $\$3.854 \times 10^{10}$, and $\$3.845 \times 10^{10}$, respectively. Thus, the cost decreases with $\rho$, which makes sense since the hour-ahead model commits more fast generation as $\rho$ increases to prevent possible shortages. The result is not as intuitive as it appears, however, because as $\rho$ increases, the decrease in shortage penalty is accompanied by higher generation costs. Then the tradeoff between more overage and less shortage becomes less obvious.
Figure 7-9: Percentage Difference by Tunable Parameter (40% Wind)

Figure 7-9 compares the percentage difference in total actual power and total actual demand for each value of the tunable parameter. More overages occur as \( \rho \) increases. Many of the additional overages also occur at the same time increments. For example, many overages for \( \rho = 3 \) are extensions of overages for \( \rho = 2 \).

Table 7-10 compares the percentage changes in shortage and overage data for the 40% simulation using the \( \rho = 1, \rho = 2, \) and \( \rho = 3 \) heuristics. Percentage changes are based on the original 40.0% simulation. No clear pattern exists for shortages: \( \rho = 2 \) reduces both the number of shortages and the total shortage by the greatest percentage, but it has the smallest effect on reducing the average shortage. The effects on overage are more straightforward. As \( \rho \) increases, the number of overages, total overage, and average overage all increase, which makes sense because the demand that must be satisfied, and therefore additional generation, increases with \( \rho \).
Table 7-10: Shortages and Overages with Tunable Parameter (40%)

<table>
<thead>
<tr>
<th>40% Simulation (excluding day 0)</th>
<th>% Change $(\rho = 3)$</th>
<th>% Change $(\rho = 2)$</th>
<th>% Change $(\rho = 1)$</th>
<th>Original</th>
</tr>
</thead>
<tbody>
<tr>
<td># shortages</td>
<td>0.45%</td>
<td>-1.1%</td>
<td>3.6%</td>
<td>445</td>
</tr>
<tr>
<td>Total shortage (MW)</td>
<td>-3.7%</td>
<td>-3.8%</td>
<td>-3.5%</td>
<td>2190000</td>
</tr>
<tr>
<td>Average shortage (MW)</td>
<td>-4.1%</td>
<td>-2.7%</td>
<td>-6.8%</td>
<td>4920</td>
</tr>
<tr>
<td># overages</td>
<td>3.4%</td>
<td>3.0%</td>
<td>2.8%</td>
<td>3376</td>
</tr>
<tr>
<td>Total overage (MW)</td>
<td>1.9%</td>
<td>1.5%</td>
<td>0.9%</td>
<td>5110000</td>
</tr>
<tr>
<td>Average overage (MW)</td>
<td>-1.4%</td>
<td>-1.4%</td>
<td>-1.9%</td>
<td>15100</td>
</tr>
</tbody>
</table>

Table 7-11 compares the percentage changes in cost distribution for each value of $\rho$. The cost of fast power increases (at a slightly decreasing rate) as $\rho$ increases. The shortage penalty decreases as $\rho$ increases, but the magnitude of percentage change experiences diminishing marginal improvement. There is an additional 1.9% decrease in shortage penalty between $\rho = 1$ and $\rho = 2$ but only an additional 0.5% decrease between $\rho = 2$ and $\rho = 3$. Despite these decreasing marginal gains, the total simulation cost is the lowest with $\rho = 3$. The tradeoff between the costs of overage and shortage, however, suggests that increasing $\rho$ will eventually increase the total cost.

Table 7-11: Cost Distribution with Tunable Parameter (40%)

<table>
<thead>
<tr>
<th>40% Simulation (excluding day 0)</th>
<th>% Change $(\rho = 3)$</th>
<th>% Change $(\rho = 2)$</th>
<th>% Change $(\rho = 1)$</th>
<th>Original Cost ($10^8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow power cost</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>116</td>
</tr>
<tr>
<td>Fast power cost</td>
<td>2.4%</td>
<td>2.0%</td>
<td>1.3%</td>
<td>68</td>
</tr>
<tr>
<td>Shortage penalty</td>
<td>-8.9%</td>
<td>-8.4%</td>
<td>-6.5%</td>
<td>219</td>
</tr>
<tr>
<td>Total cost</td>
<td>-4.5%</td>
<td>-4.2%</td>
<td>-3.3%</td>
<td>402</td>
</tr>
</tbody>
</table>
Table 7-12 compares the fast generation data for each value of $\rho$. As $\rho$ increases, the number of time periods with more fast generation than the base case also increases, suggesting a limitation of the heuristic since the additional fast generation should ideally be concentrated in fewer time periods to avoid excess generation. The percentage occurrence of outliers (time increments with 200% additional fast generation with the heuristic) stays relatively constant, but the percentage occurrence of super-outliers (time increments with 10000% additional fast generation with the heuristic) more than doubles when $\rho$ increases to 2 or 3, which is a sign of excess generation.

Table 7-12: Changes in Fast Generation with Tunable Parameter (40%)

<table>
<thead>
<tr>
<th>40% Simulation (excluding day 0)</th>
<th>$\rho = 3$ heuristic</th>
<th>$\rho = 2$ heuristic</th>
<th>$\rho = 1$ heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>% increase in fast generation over base case (10^6 MW)</td>
<td>2.5%</td>
<td>2.0%</td>
<td>1.3%</td>
</tr>
<tr>
<td># time increments with increased fast generation</td>
<td>661</td>
<td>613</td>
<td>591</td>
</tr>
<tr>
<td># (%) time increments with % change &gt; 200%</td>
<td>374 (56.6%)</td>
<td>335 (57.9%)</td>
<td>331 (56.0%)</td>
</tr>
<tr>
<td># (%) time increments with % change &gt; 1000%</td>
<td>49 (7.4%)</td>
<td>45 (7.3%)</td>
<td>20 (3.4%)</td>
</tr>
</tbody>
</table>

The above sensitivity analysis for $\rho = 1, \rho = 2, and \rho = 3$ holds true only for the 40% simulation. A sensitivity analysis on $\rho$ may be very different for the 5.2% and 20.4% simulations, which are not presented. The effect of changing the tunable parameter, in addition, depends on other characteristics of the simulation, such as the use of linear interpolation to obtain sub-hourly actual wind and demand values. As a result, the benefit of using a heuristic to increase the effective horizon of the hour-ahead model depends heavily on the assumptions of the simulation. A better idea, therefore, may be to change directly the implementation of the hour-ahead model so that the horizon is actually increased. A proposal to achieve this outcome is presented below.
7.8 Proposal to Increase the Horizon of the Hour-Ahead Model

The main takeaway from the heuristics analysis is that increasing the horizon of the hour-ahead model reduces the total cost. Different implementations of this heuristic would achieve a similar purpose, and the incorporation of tunable parameters allows for the optimization of the heuristic over many simulations. Finding a more suitable functional form of the heuristic, furthermore, may lead to even more cost reductions. For example, extra generation could be added to the demand of additional sub-hourly increments, not just the last component. Alternatively, a proportion $0 < \rho' < 1$ of extra generation could be added if future slow generation were greater than current slow generation, in case actual wind were low enough to still cause a shortage. Ultimately, however, the heuristic is merely an approximation to increase the effective horizon of the problem without changing the implementation of the hour-ahead model.

A more direct way to reduce costs, therefore, is to change the implementation of the hour-ahead model to increase its horizon. This change requires the replacement of the existing hour-ahead transition function $S^M(\cdot)$ by another function $S'^M(\cdot)$. This section proposes a lookahead policy to extend the horizon of the hour-ahead model by five minutes.

Let $T' = 13$ be the number of time increments in the new horizon. $T = 12$ is still the number of time increments per hour insofar as only 12 of the 13 solutions per decision vector are implemented: the 13th solution is used as a placeholder to gain insight
into the next hour – thereby increasing the horizon – but is not implemented. The simulation model did not previously distinguish between the number of time increments in the horizon and the number of solutions that are implemented. This proposed extension of the hour-ahead model’s horizon introduces this distinction. As a result, the simulation model established in Chapter 4 undergoes the following changes.

### 7.8.1 State Variable

The adjusted state variable at day $d$, hour $h$ is the following:

$$ S'_{d,h} = \{ S^D_d, S'^H_{d,h} \} $$

where $S^D_d$ is the state variable for the day-ahead model for day $d$ and $S'^H_{d,h}$ is the adjusted state variable for the hour-ahead model for day $d$, hour $h$.

Note that the day-ahead model and the day-ahead state variables do not change: the horizon is increased in the hour-ahead model only. It is still true, therefore, that:

$$ X^{D,\pi}(S^D_d) = x^D_d $$

$$ = [u^D_d \ y^{on,D}_d \ y^{off,D}_d \ w^D_d \ w'^{on,D}_d \ p^D_d \ e^D_d] $$

The notation for the hour-ahead state variable changes to reflect the new distinction between the length of the horizon and the length of the implemented solution. The actual computation of the state variable, however, remains the same:

$$ S'^H_{d,h} = S_{f(d,h,T-1)|T} $$

The inter-hour transition variables as defined in the hour-ahead model from Chapter 3 are still calculated at time $T$, not at time $T'$. As a result, the decisions that are scheduled to occur during time increment $T' - 1$ in the simulation are not taken into account when transitioning to the next hour $f(d, h + 1)$. The decision variables
corresponding to time $T' - 1$, therefore, are used only as placeholders but are not implemented.

### 7.8.2 Decision Variables

The adjusted decision variables consist of the day-ahead decision variables (still $x^D_d$ in matrix notation) and the adjusted hour-ahead decision variables. The latter can be written as the following:

$$x'_{d,h}^H = [u^H_{d,h} \ y'^{on,H}_{d,h} \ y'^{off,H}_{d,h} \ w^H_{d,h} \ w'^{on,H}_{d,h} \ p'_{d,h}^I \ e'_{d,h}^H];$$ the adjusted hour-ahead augmented matrix for day $d$, hour $h$ decision variables

Together, the decision variables can be written as $x'_{d,h} = \{x^D_d \ x'^H_{d,h}\}$.

### 7.8.3 Exogenous Information

The adjusted exogenous information $\bar{W}'$ consists of wind and demand. The adjusted actual wind power $p'_{d,h}^W$ for day $d$, hour $h$ is a $T' \times 1$ vector. Thus $p'_{d,h}^W$ (12), the last component of the vector, is the actual wind power for the first time increment $s = 0$ of hour $f(d, h + 1)$. Before defining this vector, several helper functions are defined. For day $d$ and hour $h$, let $\alpha(d, h)$ be a function that gives the day index of the next cumulative hour after $f(d, h)$, and let $\beta(h)$ be a function that gives the hour index of the next hour.

$$\alpha(d, h) = \begin{cases} 
    d + 1 & \text{if } h = T^d - 1 \\
    d & \text{otherwise}
\end{cases}$$

$$\beta(h) = \begin{cases} 
    0 & \text{if } h = T^d - 1 \\
    h + 1 & \text{otherwise}
\end{cases}$$

Then the adjusted wind power vector is the following:
\[
P^{W}_{d,h}(s) = \begin{cases} 
    p^{W}_{d,h}(s) & \text{if } s < T' - 1 \\
    \Omega^{\pi}(W^A, \alpha(d, h), \beta(h))(0) & \text{otherwise}
\end{cases} \quad \forall s \in [0, T' - 1]
\]

Likewise, let \( p^{L}_{d,h} \) be the adjusted total load vector with length \( T' \). Thus \( p^{L}_{d,h}(12) \) is the total load for time \( s = 0 \) of hour \( f(d, h + 1) \). Then \( p^{L}_{d,h} \) satisfies:

\[
p^{L}_{d,h}(s) = \begin{cases} 
    p^{L}_{d,h}(s) & \text{if } s < T' - 1 \\
    \Lambda^{\pi}(L^A, \alpha(d, h), \beta(h))(0) & \text{otherwise}
\end{cases} \quad \forall s \in [0, T' - 1]
\]

This method does not look into the future because the first demand value of the next hour is already used in the method of linear interpolation. If predicted values were used instead of actual values, then this method would be inadequate because it would be non-adaptive. Predicted wind and demand, however, are outside the scope of this thesis.

N.B.: An arbitrary number may replace the values \( \Omega^{\pi}(W^A, \alpha(d, h), \beta(h))(0) \) and \( \Lambda^{\pi}(L^A, \alpha(d, h), \beta(h))(0) \) for the last hour of the last day of simulation when there is no natural lookahead value.

### 7.8.4 Transition Functions

The adjusted transition function \( S^{IM}(\cdot) \) computes the adjusted state variable at the next hour from the adjusted inputs at the current hour \( h \) of day \( d \).

\[
S'_{\alpha(d,h),\beta(h)} = S^{IM}(S'_{d,h}, \{X^{D,\pi}(S^D_{d}), X^{H,\pi}(s^{H}_{d,h})\}, \overline{W}^{I}_{\alpha(d,h),\beta(h)})
\]

The first step in computing \( S'_{\alpha(d,h),\beta(h)} \) is to compute \( X^{H,\pi}(S^{H}_{d,h}) \). Recall that \( p^{S}_{d,h} \) is the total scheduled slow generation at hour \( h \) of day \( d \). Then \( p^{S}_{\alpha(d,h),\beta(h)} \) is the total scheduled slow generation at the next hour. Define \( \lambda^{H}_{d,h} \) as the adjusted total load vector of length \( T' \times 1 \) that must be satisfied by the fast generators only. Then it is calculated as follows:
\[ \lambda_{d,h}^{H}(s) = \begin{cases} \lambda_{d,h}^{H}(s) \\ p_{d,h}^{L}(s) - p_{\alpha(d,h),\beta(h)} - p_{d,h}^{W}(s) \end{cases} \quad \text{if } s < T' - 1 \]
\[ \forall s \in [0, T' - 1] \]

The hour-ahead mixed integer linear programming algorithm is not adjusted because the length of the output decision vectors is automatically the length of the input vector. Since the input \( \lambda_{d,h}^{H} \) has length \( T' \) instead of \( T \), the output augmented decision variable matrix \( x_{d,h}^{H} \) has length \( T' \) instead of \( T \). Then the output of the hour-ahead model \( X_{H}^{\pi}(\cdot) \) is:

\[ X_{H}^{\pi}(S_{d,h}, \lambda_{d,h}^{H}) = x_{d,h}^{H} \]

\[ = \begin{bmatrix} u_{d,h}^{H} y_{d,h}^{on,H} y_{d,h}^{off,H} w_{d,h}^{H} w_{d,h}^{on,H} p_{d,h}^{H} e_{d,h}^{H} \end{bmatrix} \]

The implementation of the solution must be adjusted because now not all of the components of the decision vectors are implemented. In particular, the last component corresponding to \( T' - 1 \) is not implemented. Recall that \( M(\cdot) \) is a mapping of indices such that index \( i \) in \( I^{G} \) maps to index \( M(i) \) in \( I \). After each hour \( h \) of the simulation, certain day-ahead solutions are updated by their hour-ahead counterparts via the following adjusted transition functions:

\[ u_{d}^{P}(h, M(i)) = u_{d,h}^{H}(T - 1, i) \quad , \forall i \in I^{G} \]

\[ w_{d}^{P}(h, M(i)) = w_{d,h}^{H}(T - 1, i) \quad , \forall i \in I^{G} \]

\[ p_{d}^{P}(h, M(i)) = p_{d,h}^{H}(T - 1, i) \quad , \forall i \in I^{G} \]

Note that the values on the right hand side are the second-to-last entries of each decision column vector, instead of the last entry. Only time increment \( s = 11 \) solutions are updated to the day-ahead solution because only solutions corresponding to \( s \in [0, 11] \)
are implemented. The last solution corresponding to \( s = 12 \) is scheduled by the hour-ahead model but not implemented.

The transition functions for the instantaneous decision variables, on the other hand, are not different. The reason is that the set of instantaneous hit times already looks only within the interval \( s \in [0,11] \). Recall that:

\[
S_{d,h,i}^z = \{ s \in \mathbb{Z}: s \in [0,T - 1], z_{d,h}^H(s, i) = 1 \}, \forall z \in \{y_{on}, y_{off}, w_{on}\}, i \in I^G
\]

Whatever instantaneous decision that is scheduled to occur during time increment \( s = 12 \), therefore, is ignored.

### 7.8.5 Objective Function

The objective function undergoes a slight change of notation for the updated decision matrices. The underlying idea, however, is the same since the summation occurs over \( s \in [0,11] \) and, therefore, does not take into account the time increment that is not implemented. It can be written as:

\[
\min_{\pi} \mathbb{E} \left[ \sum_{i \in I^G} \sum_{d \in [0,D^d - 1]} \sum_{h \in [0,T^d - 1]} \left( C_{d,h,i}^{fuel,G} \times p_{d}^{D,\pi}(h, i) \right) + \sum_{i \in I^G} \sum_{d \in [0,D^d - 1]} \sum_{h \in [0,T^d - 1]} \sum_{s \in [0,T - 1]} \left( C_{d,h,s,i}^{fuel,G} \times p_{d,h}^{H,\pi}(s, i) + c_{d,h,s}^{shortage} \times e_{d,h}^{H,\pi}(s, i) \right) \right]
\]

These proposed changes to the simulation model reflect a possible implementation of increasing the horizon of the hour-ahead model by a single five minute increment. They are presented here as a possible reference for further research. The extension to multiple additional increment horizons is not presented here but can be done.
by increasing the vector length of the decision variables solved by the mixed integer linear program but implementing only the initial $T$ solutions. Extending the horizon beyond 24 five minute increments may prove difficult but also unnecessary, since the increased horizon becomes less useful as it approaches double the size of the original horizon.
Chapter VIII

8 Conclusions and Extensions

This chapter reviews the conclusions of the simulations from Chapters 6 and 7 and discusses the implications of increasing RTOs’ wind penetration. The chapter closes by presenting limitations of the hour-ahead and simulation models and suggesting improvements for future areas of research.

8.1 Results and Implications

Simulations were run at 5.2%, 20.4%, and 40.0% wind penetration. The percentage difference between total actual power and demand is always correlated with wind power because less generated power is required as wind increases.

8.1.1 Increasing Wind Penetration

The simulation cost decreases by about 26.4% as wind penetration increases from 5.2% to 20.4%. Wind power is assumed free, and since wind accounts for a higher percentage of the total power, the total cost decreases. Several reasons explain why the cost decrease is not higher, given that the amount of wind is almost four times as large in the 20.4% simulation. First, the hour-ahead model does not adjust the output of the slow generators, so overages are more common with 20.4% wind. The excess slow generation is wasted whenever wind and slow generation combine to exceed demand. Second, wind power is not stable but rather extremely noisy, as shown in Chapter 5. Since large
amounts of wind tend to arrive in concentrated time increments, especially when demand has already been satisfied, the cost reduction is not as large as the fourfold increase in wind penetration would suggest.

As wind penetration increases from 20.4% to 40.0%, total cost increases by 38%. Although the additional wind power is free, the volatile nature of wind more than doubles the number of shortages. Compared to the 5.2% simulation, however, total cost increases only by 9.9%. When wind penetration increases from 5.2% to 40.0%, the proportion of cost attributable to generation decreases from 92% to 46%. Furthermore, the number of shortages increases 69%, and the total shortage increases over 500%. The large variation in wind power decreases the hour-ahead model’s accuracy in committing fast generation. Overages, on the other hand, increase in frequency and magnitude. Allowing the hour-ahead model to modify slow generation would reduce the overage and, therefore, unnecessary generation costs. Although increasing wind penetration to 40.0% provides more free power, the volatility of wind and the resulting shortage penalties outweigh the savings in generation costs. The benefits of increasing wind penetration to 20.4%, on the other hand, outweigh the shortage penalties. Thus, increasing the amount of renewable energy too soon too fast may in fact prove detrimental in preserving the efficiency of the power system. Gradually increasing the wind penetration from current levels to around 20% seems to be the better option.

8.1.2 Increasing the Horizon

Shortages tend to occur at the first time increment of each hour. Excluding day 0 in the 5.2% simulation, for example, 67% of the 264 shortages occur during $s = 0$. This tendency is a limitation of the hour-ahead model. A heuristic to increase the effective
horizon of the hour-ahead model is tested for each wind penetration simulation, and it indeed decreases total cost of the 5.3%, 20.4%, and 40% simulations by 1.7%, 2.3%, and 3.3%, respectively. There are two implications: first, increasing the horizon of the model results in better scheduling of generators, and second, the marginal benefit of increasing the model’s horizon decreases with higher wind penetration. Furthermore, improving the heuristic by using a tunable parameter improves cost reduction for the 40% simulation by an additional 1.2%.

8.2 Limitations

The hour-ahead model makes assumptions to simplify the problem and be compatible with the available data. These assumptions limit how closely the model represents the hour-ahead unit commitment problem faced by RTOs. Recognizing these limitations, therefore, is important to understanding how realistic and applicable the results are.

The major limitation of the hour-ahead model is that it uses actual wind and demand values instead of predictions. This aspect lets the hour-ahead model peek into the future at the beginning of each hour and see exactly what will happen until the end of the hour. This assumption is certainly unrealistic. The simulation results, therefore, are in a sense a best-case scenario for how well an RTO can respond to the volatility of wind and demand. Once predicted values are used and the hour-ahead model’s solutions are compared against actual values, shortages will most likely become more common because more uncertainty will be introduced to the problem.
Another limitation is the hour-ahead model’s limited horizon, which is explored in Section 6.1. The model schedules a solution to satisfy actual demand for the current hour. What happens during the next hour does not factor into the solution. In real-life, however, the horizon of an RTO’s problem will include at least some information past the horizon of its generation schedule because that information is crucial for inter-day and inter-hour transitions.

Third, the simulation assumes wind power is free. In reality, wind power incurs a marginal cost just like any other source of power, so the integration of wind into the system is not as straightforward as the simulations suggest. The day-ahead model may not automatically schedule as much wind as the forecasts suggest, and the hour-ahead model may not use all of the available wind.

Fourth, this simulation lacks a grid network. The model assumes that power from any generator in the system can be used to satisfy demand coming from anywhere in the system. It ignores the cost to transport power from source to destination. Including a grid network in the model is more realistic because it takes into account the locations of generation and usage during the optimization.

### 8.3 Extensions and Further Areas of Research

Extensions should relax the assumptions that are necessary for the hour-ahead model and the simulation model. First, the hour-ahead model should take in predicted wind and demand, which would require an algorithm for generating predictions. This extension would prevent the hour-ahead model from looking into the future at every hour.
Second, the hour-ahead model’s horizon should be extended by five minutes, and the simulation model should implement all but the last solution. Section 7.8 provides a blueprint for the increase in horizon, which can be generalized to ten or fifteen minutes. As seen in the heuristic studies from Chapter 7, this extension should reduce total costs.

Third, the hour-ahead model should represent PJM’s problem more realistically. A straightforward improvement would be to use more accurate data, such as ramp rates that are functions of output and actual generator costs instead of bid costs. Another improvement would be to allow the hour-ahead model to change the slow generation of online generators, which would decrease both overage and cost. A more complex improvement would be to implement a cool-down state and a minimum cool-down time so that generators do not transition directly between the online and off states.

Fourth, the hour-ahead model’s slack generation decision variables are the dual variables to the day-ahead model’s problem. Similarly, the day-ahead model’s slack generation decision variables provide information to the hour-ahead model. Exchanging this information between the two models would allow each to learn from its past performance and adjust its generation schedule for future performance. Such an algorithm may prove helpful in reducing total cost.

Finally, the smaller time horizon of the hour-ahead model is an appropriate setting to use battery storage. Energy dissipation is usually the limitation of batteries, but if they are used to store power from one five minute increment to the next, leakage becomes less of an issue. The prevention of unexpected shortages on the intra-hourly time scale involves a tradeoff between using storage and using expensive fast generation, so using battery storage could reduce total cost.
8.4 Final Remarks

Power generation is vital for the functioning of the U.S. economy. In today’s world, even a brownout could negatively impact the daily affairs of both businesses and individuals. Thus, PJM and other RTOs face the increasingly important problem of scheduling generation to meet forecasted demand. When considering the contemporary issue of renewable energy and the ongoing effort to integrate more wind into the system, this problem becomes even more complex. The ability of RTOs to create a day-ahead schedule that satisfies demand while minimizing total system cost depends heavily on the accuracy of wind and demand forecasts. Applying mixed integer linear programming to solve the hour-ahead unit commitment problem, therefore, is an effective way of adjusting the day-ahead schedule and improving the performance of RTOs. Additional research to incorporate short-term battery storage can take this progress further in order to improve both the dependability and the environmental friendliness of the U.S. power network.
References


