SMART-SREC: A STOCHASTIC MODEL OF THE NEW JERSEY SOLAR RENEWABLE ENERGY CERTIFICATE MARKET

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Abstract. Markets for solar renewable energy certificates (SRECs) are gaining in prominence in many states, stimulating growth of the U.S. solar industry. However, SREC market prices have been extremely volatile, causing high risk to participants and potentially less investment in solar power generation. Such concerns necessitate the development of realistic, flexible and tractable models of SREC prices that capture the behavior of participants given the rules that govern the market. We propose an original stochastic model called SMART-SREC to fill this role, drawing on established ideas from the carbon pricing literature, and including a feedback mechanism for generation response to prices. We calibrate the model to the New Jersey market, analyze parameter sensitivity, and demonstrate its ability to reproduce historical dynamics, while also inferring current expectations of future solar growth. Finally, we investigate the role and impact of regulatory parameters, thus providing insight into the crucial role played by market design.

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1. Introduction

While cap-and-trade schemes for carbon emissions have gained widespread attention in recent years as market-based tools for implementing environmental policy, an alternative approach which is now growing rapidly in many regions is the use of 'renewable energy certificates' or RECs (often called 'green certificates' or GCs in Europe). In conjunction with a government-mandated annual requirement level on renewable energy (and penalty for non-compliance), these certificates can be an effective tool to stimulate investment without the need for direct subsidies or feed-in tariffs. A certificate is simply issued to a solar generator for each MWh produced, and the generator can then sell this REC in the marketplace to a load serving entity (electric utility) that is subject to the annual requirement on the percentage of its electricity procured from renewables. If desired, these markets can be geared specifically towards encouraging growth in a particular type of renewable energy. The New

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Jersey market for SRECs (solar renewable energy certificates) provides an excellent such example, as the state of New Jersey (NJ) has witnessed dramatic growth from under 10MW of solar installations when SRECs were first issued in 2005 to over 950MW of installations by the end of 2012. (CleanEnergy (2013)) The NJ SREC market is currently the largest among about ten similar state-based SREC markets in the US, and similar markets also exist around the world including several European countries (eg, UK, Italy, Belgium, Sweden), Australia and even India. In the absence of a national carbon emissions markets in the U.S. (and given recent challenges faced by an oversupplied European carbon market), REC markets are emerging as an important policy alternative to putting a direct price on carbon, and consequently a potentially valuable tool in fighting climate change.

Understanding the dynamics of SREC prices is of utmost importance to market participants, particularly individuals or businesses considering investments in new solar installations and hoping to finance these by selling SRECs in the market. It is also crucial for utility companies to model and manage their SREC price risk, for example when deciding the timing and quantity of SREC purchases to avoid facing penalties for insufficient solar power in their fuel mix. In addition to addressing such concerns of market participants, a realistic model for SREC prices based on market structure can shed much-needed light on key policy challenges related to the design of these young and still developing markets, thus helping to ensure their continued success in the future. As has already been seen in historical prices, both cap-and-trade markets and SREC markets are intrinsically susceptible to unstable prices, which can potentially swing rapidly from nearly zero to the penalty level, despite relatively small changes in the underlying supply and demand forces. Regulators are often forced to respond to such events with rule changes or artificial market fixes, which unfortunately do not necessarily address the long-term inherent instabilities. Price instabilities can have a chilling effect on the development of a solar industry, as investors react to the risk that SREC prices will not support the investment. If states want to develop a healthy industry to encourage and support investments in solar, it is important to design markets which produce stable prices.

Despite this clear need for innovative new models to describe SREC price behaviour, there exists very little academic literature on the topic. A number of government-sponsored reports provide useful overviews of the market and some general discussion of factors affecting price dynamics. [1] Historical SREC price and issuance data are also easily available online. (see

[1]See for example Wiser et al. (2010) and Bird et al. (2011), both sponsored by the Department of Energy's National Renewable Energy Laboratory, for summaries of the development of SREC markets across the U.S. through 2011, or alternatively the New Jersey Clean Energy Program's annual reports and regularly updated news on the New Jersey market. (CleanEnergy (2013))

for example CleanEnergy (2013), FlettExchange (2013), SRECtrade (2013)) Nonetheless, there exist very few attempts to build stochastic models which describe how market rules affect price behavior. Amundsen et al. (2006) provide an early proposal for modeling green certificate prices in Europe, building on the classical commodity storage models of Deaton & Laroque (1996), Routledge et al. (2000) and others, equating the banking of certificates for future years to the storage of grains or metals. However, they do not incorporate into their work the regulatory structure of the market and its unique features. Several recent economic analyses of European green certificate markets are available, but these focus primarily on evaluating the success of existing mechanisms, comparing with feed-in tariffs, and discussing future prospects (see for example Aune et al. (2012), Haas et al. (2011), Tamas et al. (2010)). Instead, we focus exclusively on understanding price dynamics, including both historical behaviour and future scenarios. We aim to extend the research in this field by proposing a flexible new structural model for REC prices, driven by a continuous-time stochastic model for generation of renewable energy, and incorporating all the important features of regulations such as requirement level, penalty level and the ability to bank certificates for future years. Our SMART-SREC price model therefore builds directly on well-established literature on carbon emissions prices, drawing on prominent parallels between these two related environmental markets.

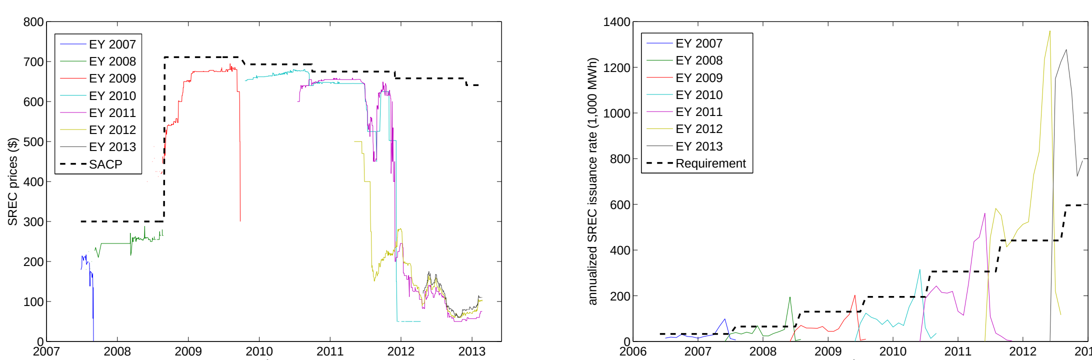
Many approaches exist for describing equilibrium price formation and dynamics in markets for emission allowances, a prominent topic in the field of environmental economics. The literature dates back several decades to early work such as Montgomery (1972) showing how cost minimization can produce an equilibrium price for a pollution credit. Rubin (1996) follows with an analysis of the inter-temporal effects of banking and borrowing credits between periods in a deterministic model. More recently, Carmona et al. (2010) present a very general stochastic framework for the behaviour of electricity and carbon market participants, leading to a single-period equilibrium allowance price given by the penalty price times the probability of a shortage of credits at the compliance date (relative to actual pollution). This same pricing formulation appears throughout other recent models (c.f. Seifert et al. (2008), Howison & Schwarz (2012), Carmona et al. (2012)) which differ in their specification of the underlying cumulative emissions process which determines the payoff of the allowance at maturity. A key modeling choice is how to incorporate the feedback of price onto emissions rate, as this can be specified as an optimal control problem for a central planner (as in Seifert et al. (2008)) or alternatively as an automatic abatement produced by the structure of the market and in particular the merit order for electricity (as in Carmona et al. (2012)). The latter is an example of the class of structural models for commodities, which seek to prescribe stochastic processes for the key fundamental factors driving price, along with an realistic but tractable transformation from supply and demand to price based on characteristics of market structure. Such approaches avoid the need for a full agent-based equilibrium approach (see for example Pirrong (2012) and Carmona & Coulon (2012) for more on applying this type of approach in various commodity markets).

For the New Jersey SREC market, we propose a structural model for price formation which mimics the equilibrium price formation for emissions allowances described in the literature above. In particular, in SREC markets as in cap-and-trade, regulated companies face a compliance deadline each year at which time sufficient credits must be submitted to avoid paying a penalty, in this case known as the solar alternative compliance payment (SACP), and typically chosen to decline in future years as solar becomes more competitive. Instead of a penalty per ton of CO2 over the cap, the penalty is now per MWh of solar energy under the requirement, which is often prescribed as an increasing percentage of total annual electricity production. Therefore, the key underlying stochastic process for SREC prices is the rate of generation of solar power, or equivalently the rate of issuance of SRECs. The uncertainty in the market has thus shifted from demand for allowances (driven by an emissions process) to the supply of certificates (driven by a generation process), with the regulator now fixing demand (requirement) instead of supply (cap). Furthermore, the emissions abatement caused by high carbon prices has a natural parallel with the interdependence between SREC prices and new solar generation. It is clear that SREC and emissions markets are very similar in spirit, an observation which we exploit in the construction of our price model.

In addition to proposing our original SREC price model, analyzing its features, and discussing the dynamic programming solution algorithm, we show that our methodology is both intuitive and successful in reproducing historical prices in New Jersey, despite an evolving regulatory landscape and frequent rule changes. In the process, we investigate what current prices reveal about expectations of solar growth rates. While several recent papers have performed empirical analyses of historical EU ETS emissions prices (c.f. Daskalakis et al. (2009), Paolella & Taschini (2008), Uhrig-Homburg & Wagner (2009)), we are unaware of any that calibrate to observed prices via a structural or equilibrium model. For the EU ETS which covers many markets, a big challenge is obtaining reliable, high frequency data on the emissions process, as well as complications like offset supply. On the other hand, for the NJ SREC market, monthly data on historical SREC issuance is a single publicly available time series (see CleanEnergy (2013)) making it an excellent candidate for testing this type of model. In addition to our main modeling and calibration contribution, we analyze various properties of the resulting price dynamics and their dependence on both regulatory parameters (like requirement level and banking), and market parameters beyond the policy maker's control (like the feedback from price onto solar generation growth rates). We show that price volatility is highly time and state dependent, and also that regulatory tools can significantly alter price behaviour, topics which should be of significant interest to policy makers, electric utilities and solar investors alike.

2. The New Jersey SREC Market

In recent years, many states in the U.S. have introduced a renewable portfolio standard (RPS) to stimulate the growth of solar and other renewable energy sources that are typically not yet competitive with other traditional fuels on a cost basis. As an integral tool for RPS compliance, many states have started REC markets, and/or SREC markets in the case of a specific solar energy target (a ‘solar carve out’). Among the now 30 states with enforceable RPS standards, 10 have set up markets to trade SRECs; Bird et al. (2011) provides a comprehensive summary of their development up through summer 2011. Among these, the New Jersey market is by far the most dominant, with the highest recorded prices so far at nearly \$700 (per MWh) and the most ambitious future requirement levels at 4.10% of the state’s electricity usage by 2028. While SREC markets in the U.S. are all relatively small and young, they are projected to grow rapidly in the near future, from around 520 MW in 2011 to around 7300MW in 2025 (Bird et al. (2011)), with about half of that total coming from New Jersey. It is interesting to compare the design of SREC markets in different states. For example, Massachusetts has implemented a form of price floor at \$300 and requirement levels which dynamically adapt to the surplus or shortage of SRECs in the previous year. However we leave such detailed comparisons for future work, and focus solely on New Jersey here.



(A) Daily average prices

(B) Monthly SREC issuance rate

Figure 1. Historical NJ SREC prices and generation for EY 2007-13

New Jersey switched in 2007 from an earlier rebate program that incentivized RPS compliance to a market-based SREC program in 2007, giving us about six years of SREC price

Energy Year	True-up Period	Oldest Rules (*no banking*) R	π	2008 change (*3-year life*) R	π	2012 change (*5-year life*) R	π
2007	3 mon	32,743	300				
2008	3 mon	65,384	300				
2009	4 mon	130,266	300	130,266	711		
2010	4 mon	195,000	300	195,000	693		
2011	6 mon			306,000	675		
2012	6 mon			442,000	658	442,000	658
2013	6 mon			596,000	641	596,000	641
2014	6 mon			772,000	625	1,707,931	339
2015	6 mon			965,000	609	2,071,803	331
2016	6 mon			115,0000	594	2,360,376	323
2017	6 mon					2,613,580	315
2018	6 mon					2,829,636	308
2019	6 mon					2,952,857	300
2020	6 mon					3,079,139	293

TABLE I. Parameters relating to historical rule changes.

history to study, as provided by Flett Exchange (FlettExchange (2013)) and shown in Figure 1a. Certificates are issued with 'vintage years', corresponding to the 'energy year' (EY) in which the electricity was produced, where EY2008 (say) refers to the 12 month period ending on May 31st, 2008. As can be seen in the price history, different vintage SRECs can trade at the same time and at different prices, although they are typically highly correlated. Price differences stem from the banking rules, which describe for how many years in the future the SREC may be used for compliance. Currently SRECs have five-year lifetimes, meaning that 2012 SRECs may still be traded until summer 2016. The exact lifetime of an SREC is slightly complicated by the so-called 'true-up period', a period of several months (currently six, from June 1st to Nov 30th) between the end of the energy year and the compliance date when load serving entities (LSEs) must submit their SRECs or pay the penalty (the SACP). It is important to note that unlike many carbon emission markets, there is banking but no borrowing from the next year in SREC markets, since future supply is a random variable anyway. Furthermore, paying a penalty this year does not imply a debt to produce an additional SREC the following year (known as 'withdrawal' in emission markets), which means from a pricing perspective that the SACPs set upper bounds on the SREC prices.

The challenge of understanding the NJ SREC price history is complicated by the fact that the rules have changed several times, including banking rules (SREC life), the SACP values, requirement values and the true-up period length. Identifying the exact dates of implementation for the numerous rule changes is difficult enough, let alone identifying the market's possibly changing expectations regarding potential new rules. Although not exhaustive, Table I summarizes the main rule changes needed to understand price history, and which we shall use for our model comparison. SREC life has been increased several times from initially one year only (no banking) up to the current five years, while SACP was originally set to \$300 for all future years, before being changed in 2008 to a gradually decreasing schedule but starting at \$711 for EY2009 (as is also shown in Figure 1a). Finally, the requirement itself has been changed several times, first from a percentage-based system originally (eg, 0.16% of total electricity for EY2009) to an absolute system (eg, 306GWh in EY2011), then back to a percentage based system (eg, 2.05% in EY2014). More importantly, the most recent rule change in summer 2012 dramatically increased the SREC requirement for 2014 (and beyond), with a jump from only 772GWh to a projected 1,633GWh.[2]

Figure 1b shows the historical annualized SREC issuance rate for New Jersey (again starting with EY2007), where annualized rate here means simply 12 times the observed monthly issuance. This allows for easier visual comparison with the annual requirements also plotted, although the relationship is masked by the strong seasonality in issuance caused by weather and daylight hours. Most years so far have been slightly under-supplied, leading to prices very near the historical SACP values, with EY2012 being the first year in which no penalty was paid. It is easy to understand why the new requirement schedule passed in 2012 was needed, given the rapid exponential growth in SREC issuance relative to the approximately linear requirement schedule, making EY2013 extremely oversupplied. Prices have responded by declining very quickly from over \$600 in mid-2011 to under \$100 in late 2012, and are currently supported by their five-year lives and uncertainty about long-term growth rates for the solar industry. In particular, it will be interesting to see how much of a slowdown in SREC generation occurs in the coming years due to the lower price regime today. Early evidence suggests a noticeable recent drop in new solar installations, with only 54.8MW in Q4 of 2012, as compared to 119.2MW a year earlier and a peak of 165.1MW in Q1 of 2012. (CleanEnergy (2013)) However, as project construction time can create a significant time lag in the response of new supply to price, it is still rather early to reliably estimate the feedback mechanism which plays an important role in our model. Nonetheless, as we shall explore in Section 6 current price levels can give us some clue about how much feedback the market is currently expecting over the coming years.

[2]Whenever requirements are set in percentage terms, we use projected numbers in MWh from Flett Exchange FlettExchange (2013). We also note that while the new rules were not officially enacted until June 2012, the proposals outlining the likely changes were publicly available in late 2011 (e.g., see history of blogs on SRECtrade (2013)), and hence likely to be 'priced in' already. In Section 6, we shall therefore use end of 2011 as the date for the rule change. An alternative choice could be a weighted average of prices under different regulatory regimes to reflect changing market expectations of the probability of rule changes, but such assumptions are likely to be rather ad hoc.

3. Model

We now introduce our stochastic model of the NJ SREC market in continuous time, with time indexed by t and measured in years. We also index energy years by $y \in \mathbb{N}$. Time $t = 0$ corresponds to the start of energy year 2007 i.e. June 1, 2006, and year $y = 1$ corresponds to the time interval $(0, 1]$.

3.1. **SREC market prices.**

SREC market prices, like prices in any other market, depend on the market demand (determined by regulations), and supply (i.e. SREC generation). In a single period model (one year and no banking), the value of an SREC at maturity (compliance date) must equal either the SACP if total generation is below the requirement, or zero if the requirement is met. This is clear by no arbitrage and forms the starting point for the analogous equilibrium pricing result to that of Carmona et al. (2010) in the carbon market setting; namely, that the price at an earlier time must be a discounted expectation of this final payoff under the appropriate risk-neutral pricing measure. As SRECs are traded assets just like CO2 allowances (with no storage and delivery costs or constraints like physical commodities), this martingale condition must also hold here to ensure no arbitrage in the market. For simplicity, we shall assume a risk-neutral world, and focus instead on the key dependencies in SREC markets which would still be valid with the addition of typical assumptions on risk premia.

The rate of SREC generation at time t is a random variable, and is denoted by g_t (MWh/year). The single-year framework described above leads to an SREC price (for energy year y) of p_t^y at time $t \in [y-1, y]$ given by

$$p_t^y = e^{-r(y-t)} \pi_t^y \mathbb{E}_t \left[1_{\{\int_{y-1}^{y} g_u \mathrm{d}u < R_t^y\}} \right],$$

where r is a constant interest rate, R_t^y and π_t^y denote SREC requirement and SACP respectively for energy year y respectively, as observed at time t,[3] and $\mathbb{E}_t[\cdot]$ represents a conditional expectation given the information set at time t. Note that at time t, $\int_{y-1}^{t} g_u \mathrm{d}u$ is known, so the expectation can be written as the probability $\mathbb{P}\{\int_t^y g_u \mathrm{d}u < C\}$ for a known constant C.

Usually SRECs are valid for k additional years after their vintage year (currently $k = 4$ in NJ), requiring an extension of the formula above. We let b_t represent the accumulated

[3]Note that we require t dependence on R and π only to capture historical rule changes (eg, for EY 2014, $R_5^8 = 772$ and $\pi_5^8 =$\$625, while $R_6^8 = 1,708$ GWh and $\pi_6^8 =$\$339 following the major 2012 legislation), however we do not explicitly model requirement levels as random variables.

number of SRECs banked from previous years[4] plus the new supply this year, defined as

$$b_t = \begin{cases} \max\left(0, b_{t-1} + \int_{t-1}^{t} g_u \mathrm{d}u - R_t^t\right) & t \in \mathbb{N}, \\ b_{\lceil t \rceil - 1} + \int_{\lceil t \rceil - 1}^{t} g_u \mathrm{d}u & t \notin \mathbb{N}. \end{cases} \tag{1}$$

Note that exactly at a compliance date ($t \in \mathbb{N}$),[5] b_t is taken to mean the remaining supply immediately following compliance, and therefore equals zero whenever the requirement is not met. Hence, the market price at time t for SRECs generated in energy year y ($y \leq \lceil t \rceil$) can be obtained by

$$p_t^y = \max_{v \in \{\lceil t \rceil, \lceil t \rceil + 1, \ldots, y+k\}} e^{-r(v-t)} \pi_t^v \mathbb{E}_t \left[1_{\{b_v = 0\}} \right]. \tag{2}$$

This formulation is similar to the multi-period carbon price formulation discussed by Hitzemann & Uhrig-Homburg (2011), with the notable exceptions that firstly SRECs cannot be banked for an indefinite number of years, and secondly 'withdrawal' is not required. As in the carbon setting, the martingale condition on prices does not necessarily hold exactly at a compliance date, since at this time a cashflow may be generated by selling an SREC to a power supplier for the SACP and then buying one back after compliance. Such a price drop at $t \in \mathbb{N}$ represents the loss of one of the $k+1$ opportunities to use the SREC for compliance, which are captured by the maximum function in (2). The equation also indicates that the price of an SREC would be equal to zero if generation is definitely greater than demand (i.e. $\mathbb{P}\{b_v > 0\} = 1$) for all the remaining years v of its validity. Hence, according to (2), price p_t^y has a lower bound of zero and an upper bound of $\max_v e^{-r(v-t)} \pi_t^v$, useful boundaries for our algorithm in Section 4.2.

3.2. **Modeling generation as a stochastic process.** To be able to compute market prices from (2), we need to know the density function for g_t (in SREC/y) for all future compliance years. Motivated by observed SREC issuance, we model g_t by

$$g_t = \hat{g}_t(p) \exp\left(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \varepsilon_t\right), \tag{3}$$

where $\hat{g}_t(p)$ represents average annualized rate of SREC issuance, which is proportional to the total installed capacity (in MW) at time t. $\hat{g}_t(p)$ is also a function of price, reflecting the intuitive behavior that industry installs more capacity when prices are higher. In addition, SREC generation varies with daylight hours and weather conditions; e.g. generation

[4]Note that we do not specifically track how much of each vintage year makes up this pool of banked certificates. As long as LSEs behave rationally by submitting their older SRECs first (i.e. as in a FIFO inventory rule), then realistically there is very little chance of SRECs expiring worthless in this pool, particularly given five-year lifetimes and growing requirement levels.

[5]For now we ignore for simplicity the 'true-up' period mentioned in Section 2, which effectively shifts the compliance date backwards by several months, but does not meaningfully change the methodology or results. For the direct comparison with historical prices in Section 5, we adjust our implementation to handle this feature, but ignore it in other sections.

in summer is expected to be more than in winter. These seasonal changes in SREC generation during a year are modeled with the sine and cosine functions in (3). Finally, uncertain changes in generation (i.e. noise) are modeled by a random variable ε_t, assumed to be stationary and independent at each time.

As mentioned in Section 2 and witnessed in recent data from New Jersey, new investment in solar generation is directly dependent on SREC market prices. We capture this effect by allowing the rate of growth of $\hat{g}_t$ at time t to be dependent on historical prices[6] up to t:

$$\frac{\ln(\hat{g}_{t+\Delta t}) - \ln(\hat{g}_t)}{\Delta t} = f(p_u^y : 0 \leq u \leq t) \quad \text{for some } f.$$

For computational tractability (discussed further in Section 4.2), we assume that dependence on price history can be captured by a single historical average price $\bar{p}_t^y$, updated via

$$\bar{p}_t^y = \delta p_t^y + (1-\delta)\bar{p}_{t-\Delta t}^y \qquad \text{and} \qquad \bar{p}_0^y = p_0^y, \tag{4}$$

where $\delta \in [0,1]$ allows for flexibility in the weighting of older versus newer price observations, and where typically $\Delta t = 1/12$ (monthly time steps). While there may be some immediate feedback effect due to some generators choosing strategically to sell more SRECs to the market when prices are highest, the majority of long-term feedback is likely to be lagged due to new project construction time which can vary significantly. In addition, different investors may look at different historical SREC price averages in order to make their investment decisions. The simple parameterization of (4) allows us to reflect this uncertainty while maintaining a low-dimensional state variable.[7]

Finally, we assume that f is simply an increasing affine function of $\bar{p}_t^y$, whereby higher prices encourage correspondingly higher rates of investment in solar:

$$\frac{\ln(\hat{g}_{t+\Delta t}) - \ln(\hat{g}_t)}{\Delta t} = a_5 + a_6\bar{p}_t, \qquad \text{for } a_5 \in \mathbb{R}, a_6 > 0 \tag{5}$$

In equation (5), a_5 captures the rate of growth independent of market price; this includes investments undertaken merely for being greener (in other words, the growth rate when prices fall to zero). Parameter a_6 represents the sensitivity of generation to the average market price.

[6]As our algorithm shall price each vintage year separately, we only use historical prices for the same vintage year when implementing this effect, a reasonable assumption given the high correlation typical between SRECs of different years.

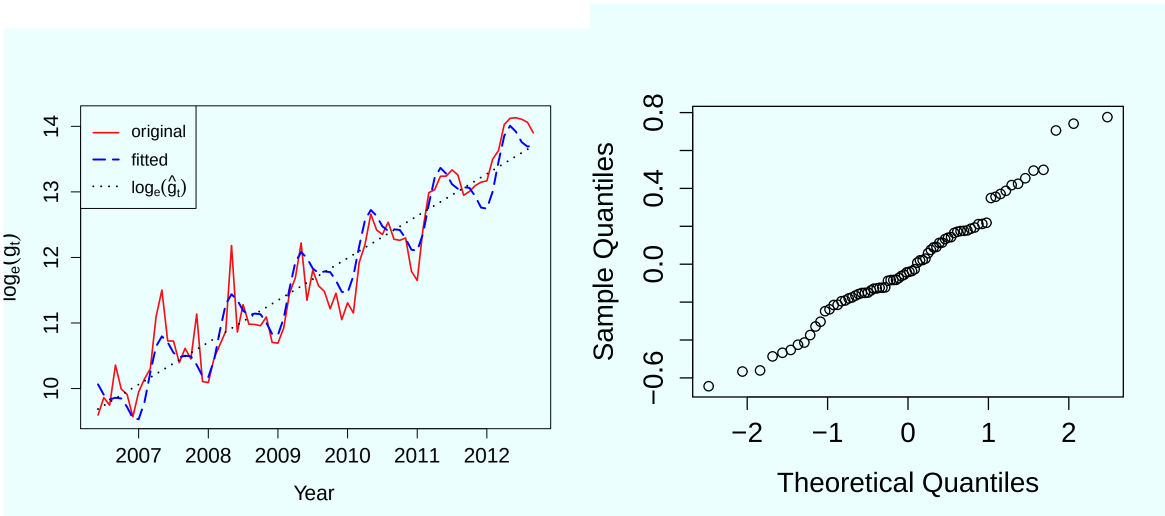
[7]A crucial advantage of this choice of price average (which produces an exponentially decaying sequence of weights for older observations), is that it is memoryless in the sense that knowledge of $\bar{p}_t^y$ is all we need in our state variable (as opposed to the entire path of p_t, as would be the case for example with a rolling window price average, or with a single lagged price point).

4. Methodology

In this section, we describe our methodology for estimating model parameters and calibrating our price model to the historical SREC prices.

4.1. Estimating parameters.

The first step is to find estimates of parameters $a_0, a_1, \ldots, a_6$ and an appropriate distribution function for ε_t.



(A) Deterministic part of the fitted function

(B) Q–Q plot for the remaining noise

Figure 2. Historical SREC generation in different energy years

The historical generation data are shown in Figure 2a. From (5) we can obtain

$$\ln(\hat{g}_t) = a_0 + a_5 t + a_6 \int_0^t \bar{p}_u \mathrm{d}u.$$

Therefore, we can rewrite equation (3) as

$$\ln(g_t) = a_0 + a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + a_5 t + a_6 \int_0^t \bar{p}_u \mathrm{d}u + \varepsilon_t. \tag{6}$$

a_0	a_1	a_2	a_3	a_4	a_5	a_6
9.576	0.058	0.184	0.346	0.236	0.646	0.000

Table II. Estimated parameters for our linear model

We estimate parameters $a_0, a_1, \ldots, a_6$ by fitting a linear model (as in equation (6)) to the historical generation data. Parameter estimates are given in Table II. Figure 2a shows both the full deterministic component (dashed blue line) and the linear trend of $\ln(\hat{g}_t)$ (dotted black line). Figure 2b shows a Q-Q plot of the distribution of noise ε_t, and suggests that it can be quite well characterized by a normal distribution. We estimate it to have a normal distribution with mean zero and variance 0.09 (i.e. $\varepsilon_t \sim N[0, 0.09]$).

Note that a_6 is near zero according to the historical data. This means that historical generation has had a constant exponential growth rate independent of SREC prices, at around 65% annually (since $a_5 = 0.646$). However, this price independence may not continue for much longer. As mentioned earlier, it can be attributed primarily to the market's youth and the high prices witnessed throughout history until very recently. Also, some generators tend to invest regardless of SREC prices, as they have other incentives such as tax benefits and reduction in their electricity bills. Furthermore some generators hedge their SREC price risk through long term contracts. Given the lack of information about likely future feedback levels, we consider a few scenarios for parameter a_6 in our analysis.

4.2. **Solving for the Price Surface.** We now discuss our methodology for solving (2) numerically for the SREC price p_t^y (i.e. at time t, for vintage year y) as a function of the state variables. In the full SMART-SREC model proposed above, the accumulated SREC total b_t, the issuance rate $\hat{g}_t$ and the average historical price $\bar{p}_t^y$ are all state variables, implying a three-dimensional array is needed for p_t^y at each time step. However, it is worth noting that by setting $\delta = 1$ in (4), we can reduce the dimensionality of the model, since p_t^y replaces $\bar{p}_t^y$ in (5). Although somewhat less realistic in practice, this simpler case of *immediate* feedback from price onto the rate of generation growth preserves the main qualitative features of the model and is hence very useful for model analysis given its faster computation time.

The central observation needed to implement a numerical solution of (2) is that discounted SREC prices satisfy the martingale condition at all time points except at compliance dates. Nonetheless, the dynamic programming algorithm requires some care, primarily because the right hand side of (2) cannot be simplified to remove dependence on p_t^y, the value which we need to solve for at each point in our discretized space. This is of course due to the feedback of price on future generation rates, which affects the calculation of each of our expectations (i.e. probabilities of ending below or above the SREC requirement in future years), and is a natural consequence of an equilibrium model for prices.

In the more general case ($\delta < 1$) the algorithm proceeds as follows:

- We first discretize space by choosing a grid of values for b_t, $\hat{g}_t$ and $\bar{p}_t$. Time is discretized in monthly steps throughout to match the frequency of historical generation data. For b_t and $\hat{g}_t$ we choose a different grid for each SREC vintage year y due to the growth of generation, with lower bounds zero throughout, and upper bounds a little above R^{y+k}, the highest relevant requirement. Similarly for $\bar{p}_t$, we choose an upper bound of $\max\{\pi^\tau : \tau = y, \ldots, y+k\}$, the highest relevant penalty.[8] Finally, we discretize the distribution of the noise term ε_t on the grid chosen for b_t.

- We next initialize the dynamic program by evaluating the payoff of the SREC at the end of its life, i.e. at $t = y + k$. At this time, all is known so no expectation appears in (2), and the price $p^y_{y+k} \in \{0, \pi^{y+k}\}$.
- We work backwards through time, solving the martingale condition at each grid point: $p^y_t = \exp(-r\Delta t)\mathbb{E}_t[p^y_{t+\Delta t}]$. Since the right hand side is bounded, continuous and decreasing in p^y_t by model construction[9], a unique solution p^y_t always exists. The system of equations (1)-(5) form a fixed point problem that can be solved iteratively via various standard root finding algorithms. (We use Matlab's 'fzero' function.)
- At times $t = y + k - i$ (for $i = 0, \ldots, k$) in the backwards dynamic program, we must incorporate the possibility that the SREC price may jump up to the SACP value if the requirement has been missed for the compliance year just ended. In other words, the martingale condition may not hold at compliance time, as can be seen in (2). This is implemented by taking the maximum of the discounted expected future value and the immediate value (the penalty times indicator of compliance today).[10]

We comment that the computation time for this exact dynamic programming algorithm can become quite large for very fine grids in all three space dimensions (i.e. for $\delta < 1$). If we choose approximately 50 grid points for each and a five year SREC life (60 time steps), this requires a few hours to solve in Matlab. If any more dimensions were to be incorporated into the model (for example, random processes for R, π or r), approximate dynamic programming approaches could be explored. However, our goal with this structural model is to suggest a sophisticated but tractable model which identifies the price's dependence on the dominant risk factors for the SREC market. Hence, we often choose $\delta = 1$ to allow for much finer space grids with shorter computation time.

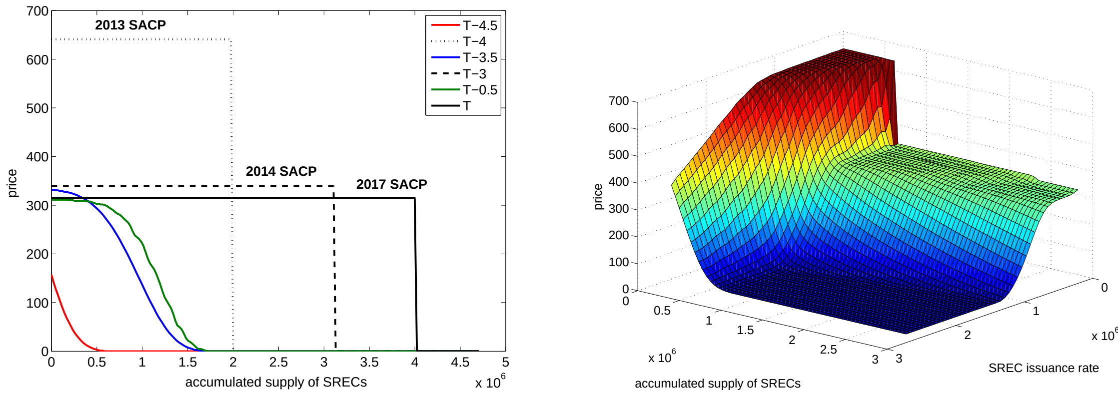
In the following sections, we first present a few illustrative results from the algorithm above, investigating the general features of the price surface solution as a function of its

[8]This is typically simply π^y since SACP is normally set by the regulator to decrease over time (as is currently the case), while requirement always increases. However, rule changes have sometimes led to different cases. Note that rules are assumed fixed for each implementation of the solution algorithm, and also that boundary conditions are straightforward to implement given the boundedness of the payoff function in (2).

[9]More specifically, this can be proven via the price bounds discussed in Section 3.1 and the fact that b_t is continuous and increasing in g_t (from (1)), while g_t is continuous and increasing in $\hat{g}_t$ (from (3)), and the drift of $\hat{g}_t$ is continuous and increasing in p^y_t (from (4) and (5)).

[10]This can be thought of as an optimal exercise decision ('value if we decide to bank for the future' versus 'exercise value for this year's compliance'), but is in fact automatic in the sense that no model is needed to make the right decision (since we assume that it is known whether the requirement has been reached). Hence, a better analogy from finance might be a stock or bond that pays a dividend or coupon only if some observable event occurs.

state variables. We then backtest the model to see how well the model predicts historical prices. While historical data is limited, these results provide some encouraging evidence that the model possesses predictive ability, as well as sensitivity to important regulatory parameters. Finally, we investigate further the importance of market design by comparing the simulated behavior of future prices while varying these policy parameters.



(A) Price curves $p_t^7(b_t)$ for different t (B) Price surface $p_t^7(b_t, \hat{g}_t)$ at $t = 7$

Figure 3. 2013 SREC results: sample price curves and surface

5. Analysis of Solution Surface

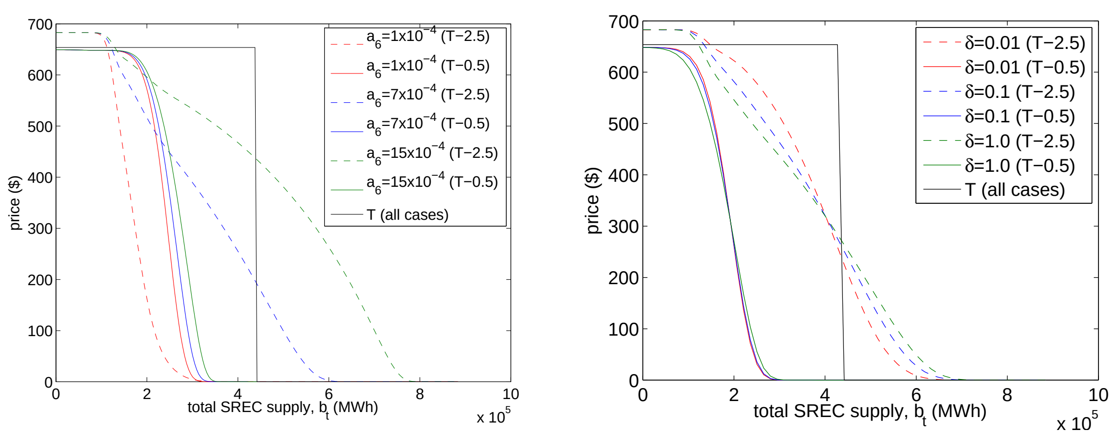
Figure 3 illustrates the typical SREC price curves generated by the model, plotted as functions of accumulated supply b_t at different times during their lives. To begin with, we choose $a_6 = 7 \times 10^{-4}$ and $\delta = 1$, representing a medium amount of price feedback on supply, with no time lag.[11] The interest rate is fixed at $r = 2\%$ throughout for simplicity, as it has very little impact on results. Time $t = y - 1$ is the beginning of the SREC's life, and we let T represent the final expiry of the certificate, which for 2013 SRECs corresponds to mid 2017 ($T = y + k$) if the true-up period is ignored, or late 2017 ($T = y + k + 1/2$) with 6 months of true-up, as assumed for Figure 3. Note also that when we are at a compliance date, we have plotted the SREC's price curve immediately *before* compliance. Hence we clearly see the discontinuity at the requirement level[12], due to the indicator function multiplied by

[11]Whenever we compare different values a_6, we also adjust a_5 such that the growth rate at a price of \$700 remains constant, and equal to our estimate for a_5 from Table II. Hence adding more feedback into the model can only slow the growth of SREC issuance, not speed it up. For example, with $a_6 = 7 \times 10^{-4}$, the growth rate has a lower bound of about 16% (for price zero) and an upper bound of 65% (for price around \$700), the original a_5 estimate.

[12]To be precise, the discontinuity in this case is above the requirement level, due to an adjustment to the methodology needed to handle a 6 month true-up period. b_t tracks the total amount of SREC issuance including new vintage year certificates which are not yet valid for compliance and hence an appropriate adjustment to R is required.

the penalty amount (SACP). If we plotted the price immediately after, it would again be smooth. Finally, as these 2D plots are in fact cross sections of 3D price surfaces $p_t^y(b_t, \hat{g}_t)$, we note that the fixed values of $\hat{g}_t$ are chosen logically to reflect the natural growth of issuance rate over time. Specifically, for plotting purposes, we choose $\hat{g}_t = \exp\{a_0 + a_5 t\}$, representing market growth at the high 'no-feedback' rate of 65% annually (continuously compounded).

In all cases SREC prices are of course decreasing in b_t, since a greater supply means a higher chance of the market meeting the requirement and not needing the SREC. Looking first at the plot for different times (Figure 3a), one striking feature here is the chance of much higher prices in the first year of the SREC's life, due to the abrupt drop in SACP from \$641 in 2013 to close to \$300 from 2014 onwards under the newest set of rules. It is interesting to note that at the first compliance date of the SREC's life (at $t = T - 4$), the price is equal to the SACP if $b_t < R_t^t$, but is not immediately zero for $b_t > R_t^t$. This right tail of the price curve implies that there is some value to banking the SREC for future periods if there is a surplus (but not an extreme surplus). On the other hand this is no longer true at the next compliance date, where the price now equals either zero or the penalty (and the same of course at $t = T$ when no more banking is allowed). While perhaps surprising at first, this makes sense because the growth rate of $\hat{g}_t$ is so much higher than that of the requirement, so even if no credits are banked at $T - 3$, the new generation is expected to be so high over the following year that the requirement is bound to be reached.



(A) Price sensitivity to a_6 (B) Price sensitivity to δ

FIGURE 4. 2010 SREC price curves $p(b_t)$ for various t, and several a_6 or δ

As we go backwards through time, it is clear that price curves diffuse to the left and flatten, as can be seen by looking at the price curves six months before each compliance date. However, since the prices can then 'jump' back up when another compliance year is reached in the dynamic program, there is no natural ordering through the five year life. The 2013 surface plot in 3b illustrates the result of diffusion in both the b_t and $\hat{g}_t$ dimensions, but for a fixed time $t = 7$, June 1st 2013, which is six months before the first compliance date (due to true-up). Prices can rise steeply towards the penalty for low values of either the accumulated SRECs or the current issuance rate. As expected, for low values of both, we see prices equal to the 2013 SACP, while for slightly higher b_t, the price drops down to the 2014 SACP value.

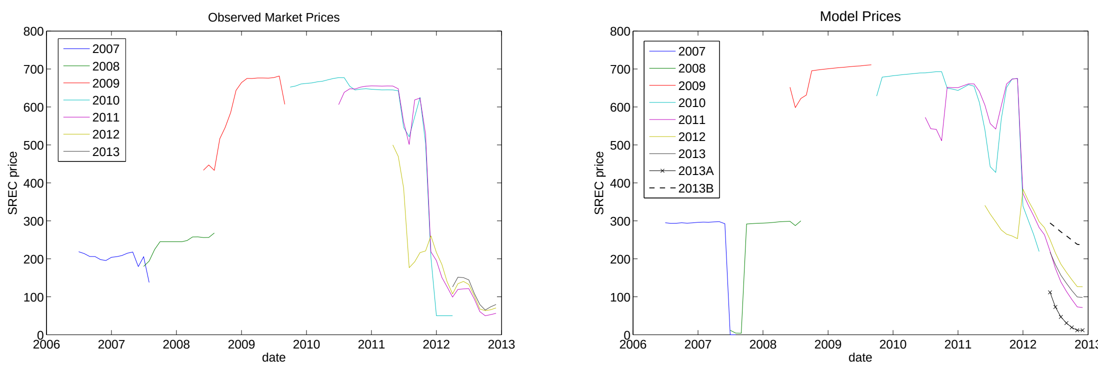
Of course these price curves are highly sensitive to the level of price feedback in the market, as well as the time lag in its occurrence. In Figure 4 we investigate these sensitivities by recalculating price surfaces for different values of a_6 and δ, this time for EY2010 SRECs (and on true-up period). In Figure 4a, we see that as the feedback parameter a_6 increases, prices increase as expected (since lower supply growth rates are now possible), but also flatten, implying that uncertainty about whether the requirement will be reached tends to last longer into the SREC's life. Furthermore, the sensitivity to a_6 is logically more significant earlier in the certificate's life (ie, at $t = T - 2.5$ in the plot) when the feedback has more time to have an effect. We choose values of a_6 corresponding to low, medium and high feedback, the last of which can actually produce negative solar growth rates if prices drop sufficiently. It is interesting to note that for the medium and high feedback cases, even if initial banked credits $b_{T-2.5}$ are above the final requirement R^{y+2}, the price $p_{T-2.5}$ is not zero. While this may seem counter-intuitive, it is in fact perfectly reasonable given that credits will be used up at the intermediary compliance years y and $y + 1$.

Finally, in Figure 4b, we investigate the impact of the time lag in the feedback effect by letting δ vary between 0.01 (very long lag), 0.1 (medium lag) and 1 (immediate, as before). Here $a_6 = 1.5 \times 10^{-3}$. Recalling the additional state variable now required, we fix $\bar{p}_t^y = \pi_t^y/2$ in the cross-sectional plots shown. The results show that delaying the feedback steepens the price curves as functions of b_t, effectively weakening the overall feedback effect. However the impact is relatively small throughout (even for several years before maturity and for a rather extreme $\delta = 0.01$), helping to justify our tendency to stick to the $\delta = 1$ case for its computational benefits.

6. Historical Price Comparison

One key criterion for assessing the model's performance should clearly be its ability to mimic observed historical price movements. Recall that, as in the spirit of traditional equilibrium or structural price models, only data on fundamental price drivers (i.e. SREC issuance, market rules, etc.) was used in our parameter estimation procedure, not prices themselves.

Hence, we should not expect to replicate a high level of detail in price dynamics, but instead for overall patterns to be consistent. In particular, with only monthly SREC generation data available, we clearly cannot predict features like daily price volatility or reaction to specific market news and announcements. Nonetheless, the structure of the model allows us to capture key long-term effects in the markets like rule changes or gradual changes in the rate of SREC issuance.



(A) Market prices (monthly averages)　　(B) Model prices

FIGURE 5. Historical comparison of market versus model prices for all years in the study. In model plot, 2013A and 2013B correspond to $a_6 = 5 \times 10^{-4}$ and $a_6 = 1 \times 10^{-3}$, respectively (while $a_6 = 7 \times 10^{-4}$ otherwise).

Figure 5 shows the results of the historical comparison for all SREC vintage years studied, with observed prices in the first plot and model prices in the second. The former are simply monthly averages of the data in Figure 1 while the latter are produced by stepping through the saved price surfaces $p(t, b_t, \hat{g}_t)$ and inputting the observed values of b_t (cumulative generation adjusted by subtracting the requirements as described by (1)) and $\hat{g}_t$ (annualized issuance approximated by using the last 12 months of observations). We set $\delta = 1$ throughout to operate in the lower dimensional case of immediate feedback. In addition, we switch between different saved price surfaces when the rules changed, as summarized in Table I earlier, which illustrates the challenge of comparing to history in this market. Finally, for the feedback parameter a_6, we set $a_6 = 7 \times 10^{-4}$ throughout, with the exception of EY2013 for which we show the impact of two alternative values of a_6 and discuss these further below.

The plots are generally encouraging, as they show that the model captures well the primary characteristics of historical price dynamics, such as the rapid price jump in 2008 associated with the first rule change, the high prices during 2009-2011, and the drop to low levels more recently. The timing of these overall movements is generally well matched, as are several details: e.g., the dip in prices in 2011 (driven by higher than expected summer and fall issuance) before a return towards the penalty at compliance (end of November in this case), and the low prices for the EY2012 certificates in this same period (due to the fact that they were not valid for 2011 compliance - no 'borrowing' allowed). Even some apparent divergence between the two plots is subtler than it seems at first. For example, in 2007, the model predicts a drop in price to zero one month before the compliance date, which is not seen in the market prices. However, in the daily prices of Figure 1, we see that the market price did drop to zero on the final day of trading. In 2007, 1,232MWh of SACPs were purchased even though the SREC issuance of 33,255MWh just slightly exceeded the requirement of 32,743MWh. (see 2009 annual report from CleanEnergy (2013)) This illustrates a detail of the market which the model is unable to capture, as there appears to be a lag in issuance information reaching all market participants, with some paying the penalty when SRECs should in theory have been available for purchase.

Indeed, several other similar details could arguably have weakened results, such as the time lag between actual solar power generation and SREC issuance. The profile of this delay appears to have changed somewhat as the market has matured, with huge spikes in issuance in June of the early years in Figure 1 becoming much less prominent in later years.[13] Many other minor factors not included in the model could potentially explain price differences, such as observable forecasts of new solar installations, unobservable market expectations (of both future issuance levels and of rule changes), and even information like changes to solar technological development and cost. Nonetheless, and despite the complexity of the market under consideration, we argue that the overall picture is very reasonable and strongly justifies our modeling approach.

Finally, we comment on the important role played by a_6, measuring the feedback, or sensitivity of new supply to price. During the majority of our period 2007-2013, this parameter has little impact on results since prices remained near the SACP. However it does significantly affect the rate of price decrease witnessed since the end of 2011, with a high a_6 producing a slower price drop because the model anticipates a significant deceleration of the solar industry's growth. To illustrate the sensitivity of results to different a_6, we include two additional lines for 2013 SRECs in Figure 5. With $a_6 = 5 \times 10^{-4}$, prices drop nearly to zero, while for $a_6 = 1 \times 10^{-3}$, they stay above \$200. Therefore, although the recent SREC *generation* history has yet to show statistically significant evidence of feedback in the market

[13]As solar generators are not required to register their SRECs immediately after actual generation, some may choose to wait until just before the compliance deadline (in the true-up period) to issue SRECs. Others may simply prefer an annual metering system to a monthly one. Finally, all SREC issuance is subject to a delay of up to a month simply due to the metering, reporting and processing time (see PJM's tracking system GATS PJM-EIS (2013) for more information).

(recall $a_6 = 0$ estimated from history), the recent SREC *price* history can provide evidence that some feedback over the coming years is expected by market participants and priced into current SREC price levels. A value for a_6 near 7×10^{-4} produces bounds on generation growth of approximately $[16\%, 39\%]$, which is still rapid growth, but significantly below the 65% seen over previous years.[14] Of course the exact value which fits best is highly model dependent. As it is evident that this parameter has key consequences on both investment and policy decisions in these markets, we shall further explore the role of this component of our model in the next section via simulations of future prices under different scenarios.

7. Policy Analysis

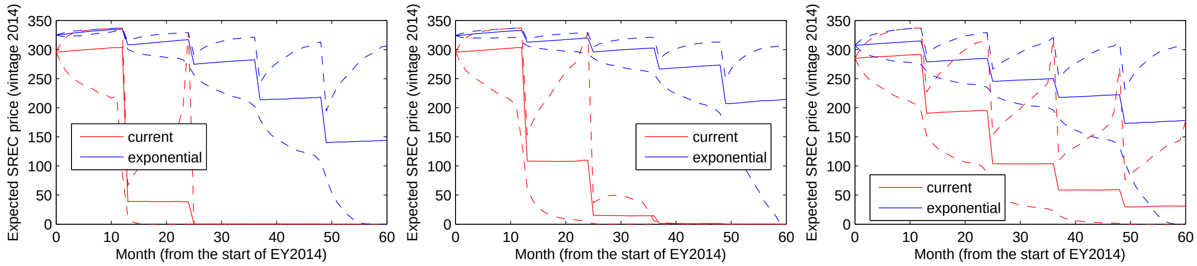
As stated before, during the past years, NJ SREC market has gone through a series of regulatory changes. These changes have been imposed because the existing regulations have not been able to keep a steady market price, attractive enough to promote investment in solar generation. In this section, we deploy the SMART-SREC model to analyse a couple of regulatory decisions faced by policy makers of SREC markets.

7.1. **Requirement Schedule.**

As can be understood from (2), requirements play a very important part in determining market prices. As shown in Figure 1, market prices fell dramatically in the beginning of 2012. This motivated the new regulation of 2012 that increased the yearly requirement greatly for EY2014, followed by a decelerating rate of requirement growth afterwards (specifically, 21.3%, 13.9%, 10.7%, 8.3% in years 2015-18, then roughly 4% thereafter, as in Table I), producing a concave future requirement schedule. On the other hand, we saw in Section 4 that generation tends to grow exponentially. In this section, we compare future market prices from the current regulation with an alternative policy imposing exponential growth on the yearly requirement. We use the alternative requirement function $\tilde{R}_t^y = R_t^8 \exp(\beta(y-8))$ for some constant β, which imposes an exponential scheme from $y = 8$ (EY2014) onwards, the relevant period for the 2014 vintage SRECs in our simulation.

Figure 6 compares the simulation results of SREC prices for the existing requirement (increasing slower than linearly by year) with our alternative exponential requirements ($\tilde{R}_t^y$). We do not try to match our initial conditions to the market today, but instead choose our values (for June 1st 2013) to be $b_7 = 0$ and $\hat{g}_7 = R^8 \exp(-\beta)$, such that in all cases by the end of the first year, the expected annualized SREC issuance rate matches the requirement. It is important to understand that a suitable choice of β in our alternative requirement schedule depends on the value of the feedback parameter a_6, as this determines the bounds on generation growth (as discussed in Section 6). We compare a low, medium and high level

[14] Notice that the upper bound is reduced significantly from 65%, simply because the SACP has been lowered. 65% growth rate only occurs when price is \$700.



(A) low feedback: $a_6 = 5e{-}04, \beta = .4$ (B) med. feedback: $a_6 = 7e{-}04, \beta = .35$ (C) high feedback: $a_6 = 9e{-}03, \beta = .25$

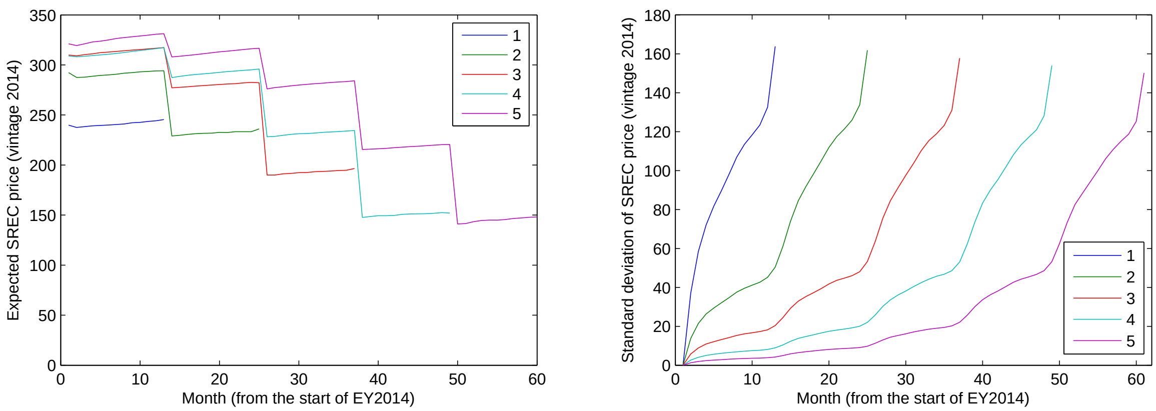
FIGURE 6. Market prices under the current requirement schedule and under the alternative of exponentially growing requirements for a 2014 vintage SREC (10th percentile, mean, and 90th percentile for 10,000 simulation runs)

of feedback, and in all cases choose β to be within these bounds, and towards the upper end of the range.

The three plots show that an exponential scheme for yearly requirements can produce long-term stable high prices with low variability provided that it is chosen appropriately to account for the level of feedback of prices on investment behavior. Under the current regulations, the results vary with feedback level, but tend to be more volatile overall, with large drops common. In particular, a very high level of feedback onto new supply is needed to prevent prices from falling towards zero over the coming years, since generation tends to grow much more rapidly than the requirement. This is especially true in the later years, as the current requirement schedule flattens out to approach an annual growth rate of only 4%. We shall use an exponentially growing requirement scheme for the rest of our policy analysis examples, as this produces more stable long-term price levels in our model and is thus more practical for the purpose of policy comparison.

7.2. **Number of Banking Years.** In this section, we use simulation to analyse the effect that the number of banking years (k) has on market prices, assessing criteria such as market stability and price level for longer banking periods.

We first consider the mean level of simulated prices when varying only the value of k, as illustrated in Figure 7a. As each additional year of validity provides an additional possibility to use an SREC for compliance, it is clear that SREC prices are always higher for higher k. In addition, the model guarantees (via the martingale condition implied by (2)) that mean prices must rise gradually with the interest rate between compliance dates and then fall suddenly at compliance dates (assuming at least some probability of missing the requirement). Note, however, that the downwards trend produced by this effect over the five-year period should not be interpreted as an overall drop in all traded SREC prices, but only in prices for the 2014 vintage. As new SRECs would be issued in later years with higher prices, the overall average price level may well be fairly constant in expectation. Alternatively, as an investor in new solar installations, one might instead be interested in the expected value of the newest SREC in each future period, which should also remain fairly constant if generation growth rates broadly match with requirement growth rates as we have chosen here.



(A) Mean of prices

(B) Standard dev. of prices

FIGURE 7. Analysis of SREC prices under different numbers of banking years: $a_6 = 5 \times 10^{-4}, \beta = 0.4$ and $\hat{g}_7 = R^8 \exp(-\beta)$ throughout. For left plot, $b_7 = 100,000$, while for right plot, b_7 varies and is chosen to obtain $p_7^8 = 150$.

A more interesting question regarding banking rules is that of price volatility. When the number of banking years increases, banked SRECs can be used for more years, which can alleviate a surplus effect and therefore prevent prices from dropping sharply. In addition, longer banking possibilities allow future shortage occurrences to be minimized, and thus prices to less often reach the SACP value. Therefore, as expected, Figure 7b shows an example in which the higher the number of banking years, the lower the standard deviation of SREC prices[15]. Note that standard deviation in this picture is only depicted for the life of the original SRECs issued, and thus the case with $k = 1$, for example, would repeat five times for the whole five years studied as new SRECs would be issued and traded each year. Therefore, this case is clearly much more volatile than, say, $k = 5$, for which banking possibilities serve to mitigate volatility in the early years. Although sharp increases in volatility can occur near intermediate compliance dates, the largest increase typically occurs near the final compliance date when SREC value reaches its binary terminal value.

[15]In order to create a fair comparison (e.g., to prevent extreme cases with standard deviation near zero due to prices equal to zero or the SACP), we have chosen the banked SREC starting point (initial value of b_t) differently for each case in order to ensure the same initial price level.

8. Conclusion

The coming years may prove to be critical ones for the future of the New Jersey SREC market, as policy makers debate tools to stabilize volatile price dynamics and to effectively encourage the growth of the solar sector. As we have seen in our analysis, SREC prices can be highly sensitive firstly to market design, including banking rules and the choice of requirement growth rate, and secondly to the behaviour of market participants such as the response of new generation to prices. Given the structural similarities to cap-and-trade markets (where price again stems from a penalty value, paid contingent on a non-compliance event), we have seen that many of the same important research questions apply, both from a pure price modeling perspective and from that of optimal policy decisions. For example, the carbon market literature has featured various studies of optimal market design, including the comparisons in Gruell & Taschini (2011) of various alternative proposals like price collars and allowance reserves, and proposals of dynamic allowance allocation schemes in Carmona et al. (2010). In the market for renewable energy credits, such issues are especially relevant research topics today, as other states and regions look to New Jersey as a guide for future REC markets of their own. In this work, we have proposed a new model, SMART-SREC, that can begin to answer such questions by understanding the key factors and relationships driving SREC prices, contributing to their volatility and sometimes surprising price swings. In particular, we have demonstrated the important role played by regulatory policy in determining price behaviour, and also analyzed the sensitivity to model-dependent parameters like feedback level which can be hard to estimate from data. Nonetheless, we have successfully calibrated to the early years of the New Jersey market, explained observed dynamics and inferred from prices a level of feedback expected by participants. Such insights illustrate an important advantage of a structural price model which exploits fundamental supply and demand relationships and market rules. Historical SREC price behavior would be very challenging to capture or understand in a reduced-form or classical econometric price model. Moreover, in order to attempt to build plausible future SREC price distributions, it is particularly critical to model the underlying regulatory structure of the markets given the high price variability and fundamental regime changes witnessed throughout their young history. We hope that our work can pave the way for a growing literature in this fascinating field of environmental economics.

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