A Stochastic Unit Commitment Model in the Presence of Offshore Wind Energy

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Abstract

This thesis examines the effects of offshore wind energy on PJM’s power generator allocation. To study this effect, the thesis will formulate a unit commitment model for PJM’s Day-Ahead Market. Then the thesis will formulate a secondary model to account for PJM’s Real-time energy rebalancing. These two models will be combined to run simulations of PJM’s energy market under the presence of power output from offshore wind farms. The simulations will output results under three levels of wind penetration: 5%, 20%, and 40%. The current model responds well to an increase to 5% wind penetration, but displays some significant load-shedding under 20% and 40% wind. However, such problems disappear completely when wind is deterministic, which shows that the main cost of wind lies in its stochasticity, and not its volatility. This result shows that with better data on wind, the model can significantly improve its results.
Acknowledgement

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To Mom, Dad, and Peter.

I dedicate this to you.
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Chapter 1

Introduction

Infrastructure development for wind power has seen a meteoric rise in the US. In the past five years, wind turbine installations have risen by a staggering 27.6%, and as of 2011, the total installed capacity of wind power clocks in at 43 Gigawatts, making the US second in the world behind China (AWEA, 2011).

![United States Wind Power Capacity](image)

Figure 1.1: US wind power capacity from 1981-2010

The upward trend illustrated in Figure 1.1 is set to continue into the future. In
the third quarter of 2011 alone, the American Wind Energy Association (AWEA) reported 90 ongoing projects that are set to add an additional 8 Gigawatts of wind power to the US energy markets. With these projects, along with developments for offshore wind energy, the US is making tremendous progress towards the 20% wind penetration benchmark that the Department of Energy has set for 2030 (DOE, 2008).

Unfortunately, great progress comes with great challenges. The main issue here is the volatility and stochasticity of wind. Wind power is subject to massive hourly fluctuations, which means that the supply of energy from wind farms is not stable. In addition, these fluctuations are stochastic, and are quite difficult to predict on an hourly basis. Because much of the power grid is not nimble enough to quickly respond to such volatility, a grid operator must have accurate wind forecasts to efficiently allocate their power generators.

One party that must consider the aforementioned issues is PJM Interconnections, one of the regional transmission organizations (RTO) in the US that operates much of the grid in the Pennsylvania-Jersey-Maryland area. With the recent influx of wind
infrastructure and the 20% wind penetration goal by 2030, it is imperative that PJM refine its generation allocation strategies to account for increased wind power. Before discussing the model that would investigate such a shift, it is important to form a basic understanding of PJM’s energy market operations and understand why PJM needs to worry about increased wind energy. In that regard, Section 1.1 will go over the procedures of PJM’s Day-Ahead Market, and Section 1.2 will detail its real-time rebalancing procedures. Section 1.3 will go over some news in offshore wind energy and how it affects PJM. Finally, Section 1.4 will give a quick overview of the rest of this thesis.

Figure 1.3: Map of PJM transmission zones, as denoted by the colored areas
1.1. PJM Day-Ahead Market

Ever since 2002, PJM has adopted an unregulated market-based bidding structure in a move to deregulate the power industry. This bidding system divides each operating day into 24 hourly blocks, and within each block, both energy suppliers and buyers submit their offers and bids for each hour of the operating day by noon on the previous day. Energy buyers and retailers, referred to as Load Serving Entities (LSEs) in the energy industry, submit demand bids that contain forecasted demand needs (referred to as ”load” in the energy industry) for each hourly block of the operating day and their maximum buying price for each block. When aggregated amongst all LSEs, these demand bids form an hourly demand curve that the energy suppliers must fill for the operating day. At the same time, energy producers submit supply offers, generator data (like maximum/minimum capacity, ramp rates, etc.), and minimum selling prices at each block (PJM, 2010). At this point, the grid operator, like PJM for example, selects generators to match the bids and offers and to minimize total generation cost. This entire process repeats daily and forms the PJM Day-Ahead Market.

Unlike a stock exchange, the auctioning process in the PJM Day-Ahead Market consists of much more than matching demand and supply. The main challenge here is that PJM must consider the physical limitations of the myriad of generator types available in the market. PJM must take into account the ramp rates, the warm-up times, and, most importantly, the fuel costs of each of the generators under its jurisdiction. For example, nuclear generators have high maximum output capacities and relatively low fuel costs. As a trade-off, however, their ramp rates are extremely slow, and their warm-up times are even slower. They possess strict minimum on/of time requirements as well; once on, they must stay on for several days to fully stabilize, and once off, they must cool down for several days before being turned on again. On the other hand, gas turbines can ramp-up and warm-up in a matter of minutes, but
they suffer from low maximum output capacity and high fuel costs.

Ultimately, generation assets can be categorized into two types: baseload and peaker generators. Baseload generators have very high output capacities and low fuel costs, but cannot ramp-up or warm-up quickly. Peaker generators cannot produce much output, but can fire up or down extremely quickly to respond to spikes or dips in demand or exogenous power output like wind. Table 1.1 illustrates this distinction.

<table>
<thead>
<tr>
<th></th>
<th>Start-up time</th>
<th>Ramp-up rate</th>
<th>Max capacity</th>
<th>Min on-off time</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>10 hours</td>
<td>15 MW/hour</td>
<td>500 MW</td>
<td>10 hours</td>
<td>$30/MW</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>5 minutes</td>
<td>50 MW/hour</td>
<td>50 MW</td>
<td>5 minutes</td>
<td>$300/MW</td>
</tr>
</tbody>
</table>

Table 1.1: Side-by-side comparison of coal and natural gas generators (PJM, 2009)

In the Day-Ahead Market, PJM faces the difficult task of managing these different generation assets and scheduling them appropriately to match the demand and supply bids at each hour. To this end, PJM formulates and solves what literature calls a *unit commitment* (UC) problem, the solution to which is an efficient generation schedule.
for the next operating day. Then for each node in the power grid, the Day-Ahead locational marginal prices (LMPs) are calculated. These are the buying and selling prices for each megawatt of power that is agreed upon in the Day-Ahead Market. PJM posts LMPs by 4 pm, and then from 4 pm to 6 pm, additional offers may be made or modified in PJM’s Balancing Market. After this point, PJM makes additional runs of its UC model to optimize next day’s generation allocation.

1.2. Real-Time Energy Rebalancing

Even after solving the UC problem, PJM still needs to tune its generator allocation in real-time to better reflect actual market conditions. Currently, the main source of uncertainty in the PJM Day-Ahead Market is the demand curve; an energy consumer’s demand bid from the previous day may not accurately reflect the actual demand needed for the current day. Figure 1.5, which plots hour-by-hour actual demand and demand forecasts from 2009 PJM data, illustrates this perfectly.

![Hourly actual/predicted demand comparison from 2009 PJM data](image)

**Figure 1.5:** Hourly actual/predicted demand comparison from 2009 PJM data
Since the Day-Ahead scheduling will map generation to the predicted demand and not the actual demand, PJM must take steps to correct the discrepancies. Once the actual demand becomes known in real-time, PJM will fire up/down its peaker generators depending on whether the demand prediction was higher or lower than the actual. While baseload generators can ramped up/down as well, they do so at a much slower rate, which makes them less suitable for real-time adjustments. In general, the generators with the cheapest fuel costs are ramped first in the act of rebalancing. However, ramp-rates must also be considered here, because the generators with the cheapest fuel costs tend to have the slowest ramp rates. Once the real-time rebalancing is complete, PJM calculates the Real-Time LMPs, which are the buying and selling prices for each megawatt of energy calculated in real-time (PJM, 2009).

The real-time adjustments will become more difficult in the presence of increased wind, since wind is a greater source of volatility and stochasticity than demand. In reality, it is difficult to predict how increased wind penetration will affect PJM’s real-time rebalancing. As of 2010, the level of wind energy constitutes only 0.5% of the total generation in the PJM grid, which means that currently, wind fluctuations have little to no affect on the system. However, this will no longer be the case as wind’s total contribution to the system increases.

1.3. Offshore Wind Energy

As stated earlier, the US is beginning to develop significant capacity for wind energy. However, from the viewpoint of a grid operator like PJM, the total capacity is not indicative of wind energy’s potential. Instead, PJM needs to consider the *locations* where wind energy is abundant. After all, grid operators must manage its transmission costs and ensure that the grid is not overloaded. Even if a stretch of land enjoys high levels of wind, the transmission costs will be too high for PJM if that location is
too far away from its main grid. In addition, transmitting electricity from a faraway location can put too much stress on the power lines, which is highly undesirable for PJM.

As seen in Figure 1.6, the Midwest is the only part of the US with significant levels of wind. However, that area serves little interest to PJM since its main jurisdiction is at the Pennsylvania-Jersey-Maryland area near the east coast. PJM’s service does extend into Illinois somewhat, as seen in Figure 1.3, but trying to transmit wind energy from there could be cost-prohibitive. Ultimately, from PJM’s standpoint, the best source of wind energy seems to be from the east coast. As evidenced by the map, the offshore wind capacity along the east coast is quite staggering.

Unsurprisingly, this potential has not gone unnoticed. The Trans-Elect Development Company has proposed an electrical transmission backbone along the east coast to tap into its significant amount of wind energy. Funded by large private investors
like Google, this project, called the Atlantic Wind Connection (AWC), is set to begin construction in 2013. Once completed, the AWC line will span 350 km along the Mid-Atlantic coast. The initial phase of the project is set to connect population and power transmission hubs in souther New Jersey and Rehobeth Beach, Delaware. When fully completed, the AWC will use 350 miles of cable as a backbone to deliver power to southern Virginia and northern New Jersey as well. Estimates place the total capacity of the AWC to be around 7000 MW, which is sufficient to power 1.9 million homes along the serviced areas (AWC 2011).

Needless to say, the injection of offshore wind will add a huge layer of stochasticity to PJM’s energy market operations. In effect, PJM will need to face stochastic elements from the supply side of its energy market as well as from the demand side. Unfortunately, wind is extremely difficult to predict accurately, which makes PJM’s UC problem even more challenging.

Other than finding accurate methods for wind prediction, an approach the grid operator could take is to find ways to smooth out the volatility of wind. One of such methods is the use of energy storage, like hydroelectric batteries. During high periods of wind, the excess generation can be pumped into storage for later use during low periods of wind. However, finding an optimal policy to implement this general idea is an entire problem class altogether. Thus, this thesis focus more on dealing with wind’s stochasticity, rather than its volatility.

If harnessed efficiently, offshore wind, because of its zero marginal cost, could help PJM and other grid operators to save generation costs. However, a UC model that does not assume wind penetration will not be able to handle the huge volatility and stochasticity of wind. It will mostly likely result in overproduction during high periods of wind, and shortages at low periods of wind. Overproduction obviously leads to unnecessary cost, and shortages will force PJM to ramp up their expensive peaker plants, which would also greatly increase cost of generation. Overall, accurate
wind predictions and a well-formed UC model are key to solving this problem.

1.4. Thesis Overview

This thesis will first summarize the main methods of solving the UC problem in Chapter 2. Chapter 3 will give a quick overview of the data that serve as inputs into the UC Model proposed in this thesis. Then, Chapter 4 will give a formulation of PJM’s UC Model, the results of which are then discussed in Chapter 5. Finally, Chapter 6 will give final remarks and comment on further areas of research.
Chapter 2

Unit Commitment Literature Review

In order to meet the demand at every hour, PJM seeks to find the optimal allocation among all of its generators. However, many generators have limitations from their start-up time, ramp-up rates, minimum on/off time, etc. To solve this problem given these constraints, PJM faces a unit commitment (UC) problem. The following sections will review some methods for solving such a problem.

2.1. Mixed-Integer Programming

Mixed integer programming (MIP) is one of the most practical and popular ways of solving the UC problem, especially given the advent of software packages like CPLEX that can solve MIPs quickly. MIPs are much like regular linear programs, but the key difference is that MIPs contain a mix of integer and non-integer variables. A sample formulation is as follows (Yan & Stern, 2002):

\[
\begin{align*}
\text{Minimize} & \quad \sum \text{Cost}(t) \cdot \text{Gen}(t) \\
\text{Subject to} & \quad \sum \text{Gen}(t) = \text{Demand} \quad \forall t \\
& \quad \text{Gen}(t) \geq 0 \\
& \quad \text{Gen}(t) \in \mathbb{Z} \\
& \quad \text{Additional Constraints} \\
\end{align*}
\]
**Decision variables**

\[ u^t_i = \begin{cases} 
1 & \text{if generator } i \text{ is on at time } t \\
0 & \text{otherwise}
\end{cases} \]

\[ y^t_i = \begin{cases} 
1 & \text{if generator } i \text{ started up at time } t \\
0 & \text{otherwise}
\end{cases} \]

\[ z^t_i = \begin{cases} 
1 & \text{if generator } i \text{ shut down at time } t \\
0 & \text{otherwise}
\end{cases} \]

\[ p^t_i = \text{Output (in MW) of generator } i \text{ at time } t \]

**Generator parameters**

\[ f_i(p^t_i) = \text{Fuel cost of unit } i \text{ at time } t \text{ with generation of } p^t_i \]

\[ p_i = \text{Minimum capacity for generator } i \]

\[ \bar{p}_i = \text{Maximum capacity for generator } i \]

\[ C_{u,i} = \text{Start-up cost for generator } i \]

\[ C_{d,i} = \text{Shut-down cost for generator } i \]

\[ R_{u,i} = \text{Ramp-up rate for generator } i \]

\[ R_{d,i} = \text{Ramp-down rate for generator } i \]

\[ U_i = \text{Minimum on time for generator } i \]

\[ D_i = \text{Minimum off time for generator } i \]
**System parameters**

\[ I = \text{Total number of generators in system} \]
\[ T = \text{Total number of time steps to simulate} \]
\[ D^t = \text{Demand at time } t \]
\[ R^t = \text{Spinning reserve requirement at time } t \]
\[ \pi^t = \text{Forecasted power price at time } t \]

**Objective Function**

The objective function for the UC problem can either minimize total costs or maximize total profits. The cost-minimizing objective is formulated below:

\[
\min \sum_{t=1}^{T} \sum_{i=1}^{I} \{ f_i(p^t_i)u^t_i + C_{u,i}y^t_i + C_{d,i}z^t_i \}
\]

The total cost is represented by the cost/MW of generation, the start-up cost, and the shut-down cost at each generator \( i \) at each time \( t \). The start-up cost \( C_{u,i} \) and shut-down cost \( C_{d,i} \) are generally constants, while the cost/MW function \( f_i(p^t_i) \) is usually modeled as a quadratic function with respect to \( p^t_i \). The quadratic function can be modeled as a piecewise linear function for the MIP.

The alternative would be to maximize total profits for the grid operator. The corresponding objective is formulated below:

\[
\max \sum_{t=1}^{T} \sum_{i=1}^{I} \{ \pi^t p^t_i - (f_i(p^t_i)u^t_i + C_{u,i}y^t_i + C_{d,i}z^t_i) \}
\]

The gross revenue for the grid operator is simply the quantity generated times the price: \( \pi^t p^t_i \), and the total cost is the same as the previous objective. An important note here is that the market price \( \pi^t \) must be forecasted one day ahead, must like the demand. This adds another layer of complexity to the problem, which may be
undesirable.

**System constraints**

1. Demand constraint:
   \[ \sum_{i=1}^{I} p_{t}^{i} = D^{t}, \forall t \in [1, T] \]
   
   This constraint stipulates that the total generation at each time step must meet the demand.

2. Spinning reserve constraint:
   \[ \sum_{i=1}^{I} u_{t}^{i} (\bar{p}_{i} - p_{t}^{i}) \geq (D^{t} + R^{t}), \forall t \in [1, T] \]
   
   This constraint stipulates that each generator leave a certain amount of reserve at each time step. Then, in the case of a demand spike, the generators will not be completely maxed out and will have some capacity to ramp up.

**Generator constraints**

1. Maximum/minimum generation constraint:
   \[ u_{t}^{i} \underline{p}_{i} \leq p_{t}^{i} \leq u_{t}^{i} \bar{p}_{i}, \forall t \in [1, T] \]
   
   This constraint sets generation limits on each generator based on its minimum capacity \( \underline{p}_{i} \) and maximum capacity \( \bar{p}_{i} \).

2. Ramp up/down constraint:
   \[ p_{t+1}^{i} - p_{t}^{i} \leq R_{u,i}, \forall i \in [1, I], \forall t \in [1, T] \]
   \[ p_{t}^{i} - p_{t+1}^{i} \leq R_{d,i}, \forall i \in [1, I], \forall t \in [1, T] \]
These constraints limits how much power generation can change from one time period to the next depending on the generator’s ramp-up rate $R_{u,i}$ and ramp-down rate $R_{d,i}$.

3. Start-up constraint:

$$u^t_i - u^{t-1}_i = y^t_i - z^t_i, \forall i \in [1, I], \forall t \in [2, T]$$

$$y^t_i + z^t_i \leq 1, \forall i \in [1, I], \forall t \in [1, T]$$

These constraints prevent generators from shutting down and starting up in the same time step, and handles the transition between shutting down and starting up.

4. Minimum on/off time constraint:

$$y^t_i + \sum_{k=t+1}^{t+U_i-1} z^k_i \leq 1, \forall i \in [1, I], \forall t \in [1, T]$$

$$z^t_i + \sum_{k=t+1}^{t+D_i-1} y^k_i \leq 1, \forall i \in [1, I], \forall t \in [1, T]$$

These constraints prevent generators from shutting down immediately after starting up, and vice versa. Once started up, the generator must stay on for minimum on time $U_i$. In the opposite case, the generator must stay shut down for minimum down time $D_i$.

These objectives and constraints complete the sample formulation of the UC problem. Other formulations include the effect of government subsidies, breakdown of power plants, disruptions in transmission, etc. The MIP model presented in this thesis will not consider these rare, random events. The model will be similar to this sample, but with some key additions and modifications.
2.2. Priority Listing

The priority listing method sorts every generator in a system with some priority heuristic, and then commits each generator in sequence to fill out the demand curve. This algorithm can be an effective way to circumvent one of the main weaknesses of the MIP: infeasibility. An MIP formulation with many constraints can have difficulties finding a feasible solution. In addition, depending on the complexity of the model, MIP formulations can take a significant amount time to output a solution. Priority lists will never run into these problems, and can yield near-optimal solutions if the sorting heuristic is reasonably formed.

A simple formulation of this algorithm is as follows (Tingfang & Ting, 2008). First, define the parameters below:

\[ P_{i}^{\text{max}} = \text{Maximum capacity of generator } i \]
\[ F_i(P_i) = \text{Fuel cost function of generator } i \text{ with output } P_i \]
\[ = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \text{ where } \alpha_i, \beta_i, \text{ and } \gamma_i \text{ are constants} \]

First, generators with the higher capacity will be given higher priority. If generators share the same maximum capacity, then the one with the lower heat rate (HR) will be given higher priority. The heat rate is calculated with the following equation:

\[ HR_i = \frac{F_i(P_{i}^{\text{max}})}{P_{i}^{\text{max}}} \]

Note that the heat rate is simply the fuel cost at a generator’s maximum capacity weighted by the maximum capacity. The heat rate, instead of the strict fuel cost, is often used in priority listing algorithms because it is the industry standard in measuring how efficiently a generator used heat energy (Tingfang & Ting, 2008). Placing priority on fuel cost alone may cause generators with low capacities to be
ranked higher than those with higher capacities. This can often lead to inefficient solutions, especially if the low-capacity generators have a non-zero minimum on/off time.

Once the generators are sorted by this priority heuristic, they are committed in sequence to fill the demand curve. When being committed, the generators are still subject to limitations like start-up time, ramp-up rate, minimum on time, etc. While the priority listing algorithm is relatively simple, its results tend to be sub-optimal when compared to the MIP, which tends to allow more complex, and thus more realistic, formulations. Due to the existence of fast solvers like CPLEX and the fact that well-formed MIPs should not be infeasible anyways, MIPs still remain one of the most popular methods of solving the UC problem. Despite this fact, this thesis will still use a modified version of this priority listing algorithm to implement its simulation of real-time energy rebalancing.

2.3. Lagrangian Relaxation

The Lagrangian relaxation method basically solves the dual of the UC problem with relaxed constraints. First, consider the demand and reserve constraints for the UC problem, reproduced below (Che & Tang, 2009):

\[
\sum_{i=1}^{I} p_t^i = D_t, \quad \forall t \in [1, T]
\]
\[
\sum_{i=1}^{I} u_t^i (\bar{p}_i - p_t^i) \geq (D_t + R_t), \quad \forall t \in [1, T]
\]

Now, the Lagrangian function \( L \) can be defined as below:

\[
L(\bar{p}, \bar{u}, \bar{\lambda}, \bar{\mu}) = \phi(\bar{p}, \bar{u}) + \sum_{t=1}^{T} \lambda_t \left( D_t - \sum_{i=1}^{I} p_t^i \right) + \sum_{t=1}^{T} \mu_t \left( D_t + R_t - \sum_{i=1}^{I} u_t^i (\bar{p}_i - \bar{p}_t^i) \right)
\]
where \( \tilde{\lambda} = (\lambda_1, ..., \lambda_T)^T \) and \( \tilde{\mu} = (\mu_1, ..., \mu_T)^T \) are the Lagrangian multipliers to the two constraints. These variables serve to relax the constraints of the original problem. Then this relaxed dual problem can be formulated as below:

\[
\begin{align*}
\max & \quad \theta(\tilde{\lambda}, \tilde{\mu}) \\
\text{s.t.} & \quad \tilde{\mu} \geq 0
\end{align*}
\]

where \( \theta(\tilde{\lambda}, \tilde{\mu}) = \min_{\tilde{p}, \tilde{u}} \left\{ L(\tilde{p}, \tilde{u}, \tilde{\lambda}, \tilde{\mu}) \middle| (\tilde{p}, \tilde{u}) \in U_i, \ i \in I \right\} \). Here, \( U_i \) is the solution space for \( \tilde{p} \) and \( \tilde{u} \) such that they satisfy all the other constraints in the UC problem, like the maximum/minimum capacity constraint, ramp-up constraint, etc.

Solving this dual problem constructs the feasible, and possibly optimal, solution \( \tilde{p}, \tilde{u} \). Lagrangian relaxation can also be used to divide a large UC problem into smaller sub-problems, which can be solved individually as above. In addition, the relaxed constraints mean that the Lagrangian Relaxation method is more likely to lead to a feasible solution than the MIP. While Lagrangian Relaxation remains a popular method of solving the UC problem, the advent of fast and reliable MIP solvers have created a tendency to formulate the UC problem as a MIP instead of a Lagrangian relaxation problem.

### 2.4. Additional Complexities

The methods covered in this chapter are perhaps the most common methods of solving the UC problem. However, the formulations given above are deceptively simple; one could possibly add additional complexities to the models to get more realistic results. One factor that could be added is thermal states. Depending on the number of hours that a generator has been on, its thermal state shifts from "cold" to "warm" to "hot". Intuitively, the start-up costs and marginal costs can be modeled as a function of these thermal states; A "cold" generator that has not been started for a long time...
will obviously take more fuel to turn on than a "hot" generator that has only been shut down for a short time.

Another thing to note is that the generator constraints given in the first subsection may not suffice as general constraints. Different types of generators may be governed by different constraints. For example, a hydro plant, along with its start-up and ramp-up constraints, may also be subject to unique water flow and balance constraints. Another exception to the constraints is the steam plant, which does not have a uniform ramp-up rate. Steam plants are attached to "fast" combustion turbines, and thus cannot start ramping up unless the turbines start up and reach a certain level of generation. At this point, the ramp-up rate of the steam plant would depend on the thermal state of the turbine, which makes modeling such generators quite complicated.

For the sake of tractability and simulation run-time, the model presented in Chapter 4 will not include many of these additional complexities. Simplifying assumptions and alternative formulations of some of these complexities will be presented in the Chapter 4 as well.
Chapter 3

Data Overview

In order to formulate the UC Model, real generator, load, and wind data are used as inputs. The generator and load data were provided by PJM and Ventyx, and the wind data was provided by Stanford University. Sections 3.1, 3.2, 3.3 detail the generator data, load data, and wind data, respectively.

3.1. Generator Data

The primary source of generator data in this thesis is the bid data provided by PJM, which provides information on generator parameters like ramp rates, warm-up times, and bid prices at hourly increments. The most important parameter to extract from this data set is the bid price for each generator. As detailed in Chapter 1, the amalgamation of the bids from energy consumers forms a cost curve for each generator. These bid curves serve as the generator cost inputs into PJM's UC Model and are updated every hour as the bids change. The model implemented in this thesis will do the same, with additional assumptions, as explained in Chapter 4. Updating the bids at every hour as such gives the allows the UC Model here to accurately reflect the dynamics of PJM's Day-Ahead Market.

While bid prices should be updated at every hour, the same logic does not apply
for physical parameters like ramp rates and warm-up times. In PJM’s Day-Ahead Market, it is possible for bids at certain hours to set certain ramp rates at 0 MW/hour, or warm-up times to 0 hours. Such glitches in the bid data occur for generators that have been on for some time; if a generator has already been on for several hours at time $t$, it is feasible for the current Day-Ahead Market to set the ramp rate and warm-up times as such at time $t$. Unfortunately, allowing this phenomenon in the UC Model would skew the results, because the slow, baseload plants may be able to ramp and warm up/down with little cost. For this reason, the physical parameters of the PJM generators are pulled from a separate data set from Ventyx. A one-to-one mapping between these two data sets was difficult to achieve, and some generators were inevitably dropped in the process.

<table>
<thead>
<tr>
<th>Generator Type</th>
<th>Steam</th>
<th>Nuclear</th>
<th>CT</th>
<th>Landfill</th>
<th>Diesel</th>
<th>Hydro</th>
<th>Max Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Ventyx</td>
<td>708</td>
<td>51</td>
<td>952</td>
<td>43</td>
<td>74</td>
<td>82</td>
<td>294073.1</td>
</tr>
<tr>
<td>After Ventyx</td>
<td>525</td>
<td>47</td>
<td>738</td>
<td>1</td>
<td>6</td>
<td>10</td>
<td>233351.283</td>
</tr>
</tbody>
</table>

Table 3.1: Tally of generation assets before and after Ventyx integration

Figure 3.1: Comparison of generation mixture before and after Ventyx integration
While the difference in total number of generators seems rather serious, the percent difference in total maximum capacity turns out to be around 20%, which is within an acceptable range. Furthermore, as evidenced by Figure 3.1, the distribution of the total generation capacity is quite similar in both cases. Since the updated generator parameters of the Ventyx data are more accurate than those of the PJM bid data, this integrated data set will be used in the UC Model presented in this thesis.

3.2. Demand Data

For this thesis, PJM provided two sets of demand data: a time series of the aggregate demand along with a corresponding time series of demand predictions, and a set of historical demand at each node of the transmission grid. The aggregate version of the data can be seen in Figure 1.5 of Chapter 1. The node-by-node data, when aggregated, forms a similar demand curve, shown in Figure 3.2.

![Figure 3.2: Node-by-node demand: aggregated](image)

To stay truer to PJM’s grid constraints, the node-by-node data set will be used as the demand input in this thesis. In addition, using the node-by-node data set allows
the model to filter out nodes that did not map to the current generator data set or nodes with corrupted data. The unfortunate side effect is that prediction data was not available in a node-by-node basis, which means that the aggregate demand must be deterministic in this model. With the other data set, the discrepancy between the actual demand and predicted demand stayed close to around 10%. This thesis will assume this error margin small enough that assuming deterministic demand will not skew the results significantly. This leaves wind as the main source of stochasticity in this thesis. Wind is significantly more volatile and stochastic than demand, which also supports the deterministic assumption.

### 3.3. Wind Data

The wind data for this thesis was provided by Stanford University, and consists of 6 offshore wind farms off the northeast coast of the US. More specifically, the locations represented are Martha’s Vineyard, Block Island, Long Island, and Nantucket. The other 2 offshore wind farms are simulated data based on the wind from these 4 locations. The current wind data set contains the output (in MW) of each wind farm at every hour in January, 2010 for one month. Figure 3.3 on the next page displays the output of each farm at each hour.
Figure 3.3: Outputs of 6 offshore wind farms on the US east coast
Because these sample wind farms are clustered in the northeast coast of the U.S, one may notice that the wind time series for all of them move quite similarly. In other words, all 6 wind outputs tend to spike and dip at the same time. The side effect of this trend is that the aggregate wind in the UC Model is much more volatile. One way to offset aggregate wind volatility would be to pick more wind farms from different locations, but this is not possible given the current wind data set.

The 3 levels of wind penetration explored in this thesis will be 5%, 20%, and 40%. To calibrate the wind data for these cases, the outputs of the 6 wind farms are aggregated, and then compared against the total load in a simulation run. The percentage of the total load that comes from wind output is then set as the wind level. To achieve different values for the wind penetration, the aggregate wind was scaled by a constant in each case. The results of this scaling can be seen in Figure 3.4.

![Figure 3.4: Aggregate wind output](image)

In this thesis, these aggregated wind time series are then used to make day-ahead...
predictions for the Day-Ahead UC Model. Wind predictions for hour $t$ of the next day is derived from the wind output at the corresponding hour from the previous seven days. At each hour, the $q$ quantile of the wind at the same hour for the past 7 days is set to be the predicted wind. Figure 3.5 compares the wind prediction and the actual wind in the 20% wind case for 3 values of $q$: 0.3, 0.5, and 0.7.

![Figure 3.5: Wind predictions at different values of $q$](image)

As observed here, increasing the quantile value $q$ tends to increase the wind predictions. Considering that the predicted wind is an input value to the Day-Ahead UC Model, $q$, in one sense, serves as the measure of the model’s ability to utilize wind. To restrict the Day-Ahead Model’s ability to use wind, one could pick a low value for
\( q \), and vice versa if one wished to allow the model to use more wind. In Chapter 5, the value of \( q \) will be tuned over several runs to minimize total simulation costs. As it turns out, tuning \( q \) also proves to be vital for eliminating shortages as well.
Chapter 4

The Unit Commitment Model

This chapter will present a unit commitment model to solve PJM’s power allocation problem. The proposed solution consists of two steps: the Day-Ahead model and the Hour-Ahead model. The following sections will propose the mathematical formulation of these two components.

4.1. The Day-Ahead Model

The Day-Ahead model will solve for an optimal generator allocation for the next day with predictions serving as placeholders for actual demand and wind data. Some of the constraints for this model will be taken from the sample formulation in Chapter 2, with several additions and modifications to account for the different assumptions and data.

4.1.1 Model Assumptions

Modeling every intricacy in PJM’s Day-Ahead Market would result in an intractable and most likely infeasible MIP. To avoid such problems, assumptions are necessary to simplify the model. The following assumptions allow for a simpler model without
sacrificing the accuracy of the results.

1. The first and perhaps most important simplification made in this model is the calculation of bid costs for each generator. As explained in Chapter 1, the amalgamation of demand bids form a cost curve for each generator, an example of which is shown in Figure 4.1.

![Sample cost curve for a steam plant](image)

Figure 4.1: Sample cost curve for a steam plant

Unfortunately, not all cost curves can be modeled as piece-wise linear function. In addition, going through the piece-wise process for all of the PJM generators would increase the runtime to unmanageable levels. Thus, this model uses an averaged cost of operations for each generator. Let $c(x)$ be the bid curve. In addition, let $p_{min}$ and $p_{max}$ be the minimum and maximum capacity of the generator, respectively. Then we calculate the average cost $c_{avg}$ as follows:

$$
c_{avg} = \frac{\int_{p_{min}}^{p_{max}} c(x)dx}{p_{max} - p_{min}}$$
This averaging scheme is reasonable given the monotonically increasing nature of most generator cost curves. In addition, if a generator has a horizontal bidding curve, this equation will still yield the corresponding constant cost.

It is important to note that this scheme is only used to solve the Day-ahead MIP. When tallying total costs at the end of a simulation trial, the actual generation cost (a.k.a. the fuel cost of generation) will be used. Using the actual generation cost for the final cost calculation is much more accurate because for many baseload generators like nuclear plants, it is common for the bid costs to be 0.

2. This model does not differentiate between thermal states ("hot" and "cold") of generators, as explained in the previous section. This difference noticeably affects only the baseload generators when they are turning on, and in most power grid operations, this is generally a one-time event since baseload generators usually do not turn off after turning on because of the time constraints. Thus, eliminating this distinction saves runtime in running the MIP without affecting the results significantly.

3. This model does not consider generator start-up costs when allocating output. The main reason for this is that start-up costs depend on the current thermal state of the generator, which this model will not include. The marginal cost (in $/MW) of running a generator greatly exceeds its start-up cost, especially when a generator is being run for an extended period of time, so this assumption most likely will not have a significant impact on the results.

4. All of the renewable energy assets in the model (hydroelectric plants, wind farms, etc) are assumed to have zero marginal cost. The marginal cost calculations for renewable assets become extremely complex because their costs rely heavily on atmospheric conditions like rainfall, temperature, river flow, etc.
Predicting these conditions is outside the scope of this paper. Fortunately, compared to the cost of running non-renewable assets like coal plants, the marginal costs of renewable energy is quite negligible.

5. This model does not consider supply-side forces in the PJM energy markets. A major part of PJM’s operations is to match energy buyers with willing energy suppliers. Unfortunately, data critical for modeling the supply-side, like minimum sell prices and past offer history, are some of PJM’s most confidential information. To simplify matters and overcome this lack of data, this model will assume that PJM can manipulate any generator necessary to fill out the aggregated demand curve for the day.

6. In this model, the nuclear generators will start at their on states and remain on for the entirety of the simulations. This assumption is made because many of the nuclear generators in the PJM bid data did not map to the new Ventyx data set, which means that their ramp rates and warm-up times are undefined. While most generators that did not map to the Ventyx data were dropped, the nuclear generators produce far too much baseload generation to be merely dropped from the model. Since the nuclear generators cannot ramp up/down very quickly anyways, it is reasonable for them to merely stay on for the entirety of the simulation.

4.1.2 MIP Variables and Constraints

The list of variables that will be used in this UC model is listed in the subsequent sections. Before moving onto the model, however, it is important to explain the notation used for all the variable definitions.

Ultimately, the model presented in this chapter is an information process with a
Hence, all decision variables will be denoted with the following notation:

\[ x_{t,t',i} = \text{Decision about generator } i. \text{ Decision made at time } t. \text{ Executed at time } t'. \]

In its normal operations, PJM runs its day-ahead UC model at noon to make decisions for the next day. So in the notation above, \( t \) will always be noon of the current day, while \( t' \) will be some hour between noon and midnight of the next day. In this model, the times denoted by \( t' \) will be indexed 1, ..., \( T \), where \( T \) is the number of time periods to solve for the next day (24 hours in this case). The subscript \( i \) will be indexed 1, ..., \( I \), where \( I \) is the number of non-renewable energy assets (coal generators, steam plants, etc) in the system. Wind farms will be treated separately since they cannot be directly controlled. Whenever wind farms are involved, the decision variable will have the subscript \( j \in [1, J] \), where \( J \) is the total number of wind farms in the system.
4.1.2.1 Decision Variables

The variables listed below will be the ones being directly manipulated by the MIP model to optimally allocate power generation.

\begin{align*}
    u_{t,t',i} &= \begin{cases} 
        1 & \text{if generator } i \text{ is on at time } t' \\
        0 & \text{otherwise} 
    \end{cases} \\
    y_{t,t',i}^{on} &= \begin{cases} 
        1 & \text{if generator } i \text{ was turned on at time } t' \\
        0 & \text{otherwise} 
    \end{cases} \\
    y_{t,t',i}^{off} &= \begin{cases} 
        1 & \text{if generator } i \text{ was turned off at time } t' \\
        0 & \text{otherwise} 
    \end{cases} \\
    w_{t,t',i}^{on} &= \begin{cases} 
        1 & \text{if generator } i \text{ began warming up at time } t' \\
        0 & \text{otherwise} 
    \end{cases} \\
    w_{t,t',i} &= \begin{cases} 
        1 & \text{if generator } i \text{ is warming up at time } t' \\
        0 & \text{otherwise} 
    \end{cases} \\
    p_{t,t',i} &= \text{MW of power generated by generator } i \text{ at time } t' \\
    p_{t,t',j} &= \text{MW of power generated by wind farm } j \text{ at time } t'
\end{align*}

The important distinction here is warming up a generator vs. turning on a generator. A generator must go through a warm-up period before it can actually turn on to minimum capacity. Once turned on, the generator may begin ramping up its output to reach the desired level.
4.1.2.2 Generator Parameters

These variables serve as mathematical descriptors for each generator.

\[ p_{\text{min}}^i = \text{minimum output capacity of generator } i \]
\[ p_{\text{max}}^i = \text{maximum output capacity of generator } i \]
\[ \tau_{\text{on}}^i = \text{minimum on time for generator } i \]
\[ \tau_{\text{off}}^i = \text{minimum off time for generator } i \]
\[ \Delta_{\text{up}}^i = \text{ramp-up time for generator } i \]
\[ \Delta_{\text{down}}^i = \text{ramp-down time for generator } i \]
\[ C_i = \text{marginal cost of operation for generator } i \text{ (in } \$/\text{MW}) \]

These parameters will be drawn from PJM’s generator data. To find \( C_i \), a bid curve averaging scheme, as explained in Section 4.1.1, will be used to find a constant operation cost for a given day. As such, \( C_i \) is the only generator parameter that changes from day to day; the rest of the parameters should be constant throughout the entire simulation.

4.1.2.3 End-of-day Generator Parameters

These variables denote the state of each generator at the end of a day. These parameters are carried over into the next day’s MIP to allow for reasonable transition in
generator states.

\[ p_{t,i} = \text{output of generator } i \text{ at the end of a day (in MW)} \]
\[ u_{t,i} = \text{on/off status of generator } i \text{ at the end of a day} \]
\[ n_{t,i} = \text{number of hours that generator } i \text{ has been on at the end of a day} \]
\[ m_{t,i} = \text{number of hours that generator } i \text{ has been off at the end of a day} \]
\[ \phi_{t,i}^{on} = \begin{cases} 1 & \text{if generator } i \text{ satisfies minimum on-time requirement at the end of a day} \\ 0 & \text{otherwise} \end{cases} \]
\[ \phi_{t,i}^{off} = \begin{cases} 1 & \text{if generator } i \text{ satisfies minimum off-time requirement at the end of a day} \\ 0 & \text{otherwise} \end{cases} \]
\[ w_{t,i} = \text{warm-up status of generator } i \text{ at the end of a day} \]
\[ o_{t,i} = \text{number of hours that generator } i \text{ has been warming up at the end of a day} \]
\[ \psi_{t,i}^{warm} = \begin{cases} 1 & \text{if generator } i \text{ satisfies minimum warm-up requirement at the end of a day} \\ 0 & \text{otherwise} \end{cases} \]

These parameters are used in the next day’s MIP to construct transition constraints for the first hour.

**4.1.2.4 System variables**

These two variables represent the system components in this model.

\[ D_{t,t'} = \text{total predicted demand to meet at time } t' \]
\[ p_{t,t',j}^{wind} = \text{predicted wind at time } t' \text{ at wind farm } j \]
\[ \epsilon_{t,t'} = \text{slack term for the objective function} \]

The total predicted demand values (\( D_{t,t'} \)) are acquired from PJM data, and the
predicted wind data \( (p_{t,t',j}) \) will be provided by a climate group at Stanford University. The slack term \( (\epsilon_{t,t'}) \), as seen in the objective function, is added to allow for the total generation to be less than the total required demand in periods of low wind or high demand without crashing the MIP. This term is penalized heavily to ensure the optimality of the final solution.

4.1.2.5 Objective Function

This formulation will use a simple objective function for cost-minimization, as detailed below:

\[
\min \left\{ \sum_{t'} \sum_i C_i \cdot p_{t,t',i} + C_{\text{under}} \cdot \epsilon_{t,t'} \right\}
\]

As detailed in previous sections, \( C_i \) is the cost/MW of running generator \( i \), and \( p_{t,t',i} \) is the power output of generator \( i \) at time \( t' \). \( \epsilon_{t,t'} \) is the amount by which the total output is below the total demand at time \( t' \). This term is penalized accordingly with the weight \( C_{\text{under}} \), which is the cost of underage (in $/MW).

4.1.2.6 Generator Constraints

The following constraints for the MIP model physical constraints for the generators, like warm-up time, maximum output, etc. In this section, \( I \) will denote the total number of generators in the system, and \( T \) will denote the number of hours in the optimization horizon (usually 24 hours in the UC problem).

1. Capacity constraint

\[
\begin{align*}
    p_{t,t',i} & \geq p^{\text{min}}_i \cdot u_{t,t',i}, & \forall i \in [1, I], t' \in [1, T] \\
    p_{t,t',i} & \leq p^{\text{max}}_i \cdot u_{t,t',i}, & \forall i \in [1, I], t' \in [1, T]
\end{align*}
\]
These constraints ensure that $p_{t,t',i}$, the output of generator $i$ at time $t'$, stays within the output bounds of generator $i$, as defined by the maximum capacity $p_i^{\text{max}}$ and minimum capacity $p_i^{\text{min}}$.

2. On/off constraint

$$y_{t,t',i}^{\text{on}} + y_{t,t',i}^{\text{off}} + w_{t,t',i}^{\text{on}} \leq 1, \quad \forall i \in [1, I], t' \in [1, T]$$

$$u_{t,t',i} - u_{t,t'-1,i} = y_{t,t',i}^{\text{on}} + y_{t,t',i}^{\text{off}}, \quad \forall i \in [1, I], t' \in [1, T]$$

The first constraint here ensure that a generator cannot turn on, turn off, and begin warming up within the same hour, and the second constraint ensure that the decision variables are updated accordingly when a generator turns on or off.

3. Minimum on/off time constraint

$$y_{t,t',i}^{\text{on}} + \sum_{t''=t'+1}^{\min(t'+\tau_{i}^{\text{on}}-1,T)} y_{t,t'',i}^{\text{off}} \leq 1, \quad \forall i \in [1, I], t' \in [1, T-1]$$

$$y_{t,t',i}^{\text{off}} + \sum_{t''=t'+1}^{\min(t'+\tau_{i}^{\text{off}}-1,T)} w_{t,t'',i}^{\text{on}} \leq 1, \quad \forall i \in [1, I], t' \in [1, T-1]$$

These constraints ensure that a generator does not stay on/off for more than the allotted minimum on/off times ($\tau_{i}^{\text{on}}$ and $\tau_{i}^{\text{off}}$). These constraints only apply if $\tau_{i}^{\text{on}} \geq 1$ and/or $\tau_{i}^{\text{off}} \geq 1$. Otherwise, these constraints will not apply to generator $i$. Note that for the second constraint, $w_{t,t'',i}^{\text{on}}$ is used instead of $y_{t,t'',i}^{\text{on}}$.

This formulation states that a generator cannot begin warming up until it has been off for at least $\tau_{i}^{\text{off}}$ hours. Since a generator must warm up from its off state before turning on, this effectively has the intended effect of making sure that the a generator cannot turn on for at least $\tau_{i}^{\text{off}}$ hours after it turns off.
4. Warm-up constraint

\[
\begin{align*}
&w_{t,t',i} + \min\{\tau_{\text{warm}} + T - 1, \tau_{\text{on}}\} \\
&+ \sum_{t'' = t'}^{t'' = t'' + 1} y_{t,t''} \leq 1, \quad \forall i \in [1, I], t' \in [1, T - 1] \\
&0 \leq u_{t,t'} + w_{t,t',i} \leq 1 + y_{t,t',i}, \quad \forall i \in [1, I], t' \in [2, T] \\
&(1 - u_{t,t',i} - w_{t,t',i}) - (1 - u_{t,t'-1,i} - w_{t,t'-1,i}) = y_{t,t',i} - w_{t,t',i}, \quad \forall i \in [1, I], t' \in [2, T]
\end{align*}
\]

The first constraint here is exactly the same as the minimum on/off time constraint, with the minimum on/off period replaced by the warm-up period, \(\tau_{i}^{\text{warm}}\). This constraint only applies when \(\tau_{i}^{\text{warm}} > 0\). The second constraint handles the transition between the warm-up phase and the operational phase and is much like the on/off constraint mentioned earlier. The third constraint ensures that generator \(i\) cannot produce any output during its warm-up phase. The fourth constraint is analogous with the second on/off constraint presented earlier; it ensures a smooth transition between a generator’s off state and its warming-up state.

5. Ramping constraint

\[
\begin{align*}
p_{t,t',i} &\leq p_{t,t'-1,i} + \Delta_{i}^{up} + (p_{i}^{\text{min}} - \Delta_{i}^{up}) \cdot y_{t,t',i}, \quad \forall i \in [1, I], t' \in [2, T] \\
p_{t,t',i} &\geq p_{t,t'-1,i} + \Delta_{i}^{down} - M \cdot y_{t,t',i}, \quad \forall i \in [1, I], t' \in [2, T]
\end{align*}
\]

The first constraint ensures that a generator cannot ramp-up greater than allowed by it’s innate ramp-up rate, \(\Delta_{i}^{up}\). The only exception in this constraint is when a generator turns on. When a generator exits the warm-up phase and becomes operational, its output must immediately jump from 0 to the minimum capacity, \(p_{i}^{\text{min}}\). This exception is reflected mathematically in the first constraint.
The second constraint follows the same idea, but covers the case of ramp-down. In this case, the exceptional event that must be handled is when a generator turns off. When a generator turns off, it is unplugged from the grid, and no longer contributes any output. In mathematical terms, $p_{t,t',i}$ must immediately go to 0 in this case. To do this, let $M$ be an arbitrarily large number, as expressed above. Then when a generator turns off $y_{t,t',i}^{off}$, the ramp-down constraint is slackened to allow this sudden output drop to occur.

4.1.2.7 End-of-day Transition Constraints

The following constraints apply only for the first hour of a day. They ensure that the end-of-day parameters from the previous day are taken into account when starting a new MIP for the current day.

1. **Generator on/off transition constraint**

   \[ u_{t,1,i} - u_{t,i} = y_{t,1,i}^{on} - y_{t,1,i}^{off}, \quad \forall i \in [1,I] \]

   This is the exact same constraint as expressed in the previous section, except at $t' = 1$ and with the generator’s on/off status from the previous day, $u_{t,i}$.

2. **Minimum on/off time transition constraint**

   \[
   \begin{align*}
   \phi_{t,i}^{on} + \sum_{t''=1}^{\tau_{i}^{on}-n_{t,i}T} y_{t,t'',i}^{off} & \leq 1, \quad \forall i \in [1,I] \\
   \phi_{t,i}^{off} + \sum_{t''=1}^{\tau_{i}^{off}-m_{t,i}T} w_{t,t'',i}^{on} & \leq 1, \quad \forall i \in [1,I]
   \end{align*}
   \]

   These constraints are the same minimum on/off time constraints as expressed in the previous section, except at $t' = 1$. $n_{t,i}$ is the number of hours that a
generator has been on if it is still on at $t' = 1$, and $m_{t,i}$ is the number of hours that a generator has been off if it is still off at $t' = 1$.

3. Ramping transition constraint

$$p_{t,1,i} \leq p_{t,i} + \Delta_{i}^{up} + (p_{i}^{min} - \Delta_{i}^{up}) \cdot y_{t,1,i}^{on}, \quad \forall i \in [1, I]$$

$$p_{t,1,i} \geq p_{t,i} + \Delta_{i}^{down} - M \cdot y_{t,1,i}^{off}, \quad \forall i \in [1, I]$$

These constraints are the same as the ramping constraints during the day, except at $t' = 1$. Here, $p_{t,i}$ is the output of generator $i$ at the end of the previous day.

4. Warm-up transition constraints

$$\psi_{t,i}^{warm} + \sum_{t''=1}^{\min(\tau_{i}^{warm} - o_{t,i}, T)} y_{t,t''}^{on,i} \leq 1, \quad \forall i \in [1, I]$$

$$w_{t,1,i} - w_{t,i} = w_{t,1,i}^{on} - y_{t,1,i}^{on}, \quad \forall i \in [1, I]$$

$$(1 - u_{t,1,i} - w_{t,1,i}) - (1 - u_{t,i} - w_{t,i}) = y_{t,1,i}^{off} - w_{t,1,i}^{off}, \quad \forall i \in [1, I]$$

These are the same warm-up constraints as expressed in the previous section, but at $t' = 1$. The third warm-up constraint in the previous section does not need a corollary here because it already covers the case of $t' = 1$.

4.1.2.8 System Constraints

The following constraints for the MIP model the constraints to the system as a whole.

1. Demand constraint

$$\sum_{i} p_{t,t',i} + \epsilon_{t,t'} \geq D_{t,t'}, \quad \forall t' \in [1, T]$$
Here, $D_{t,t'}$ denotes the total energy demand (in MW) to be satisfied at time $t'$. $\epsilon_{t,t'}$ is an error term that allows this constraint to be satisfied even when the total output is less than the total demand. With the increased presence of randomness from wind, it is reasonable that the total output at time $t'$ may be less than the amount needed. This error term prevents the MIP from becoming infeasible in this case. To prevent this from undermining the solution’s optimality, it is penalized accordingly in the objective function.

2. **Reserves constraint**

$$\sum_i (p_{i,t,t'}^{\text{max}} \cdot u_{t,t',i} - p_{t,t',i}) \geq \rho \cdot \max(D_{t,t'}), \quad \forall t' \in [1, T]$$

This constraint dictates that PJM’s generators cannot all be operating at maximum capacity. Instead, they must aggregately leave some capacity in reserve to handle emergency demand spikes or dips in wind. The reserve capacity here is specified by $\rho \cdot \max(D_{t,t'})$. As specified by the North American Electric Reliability Corporation (NERC), grid operators must leave 1% of peak demand forecasts in reserve at all times. In mathematical terms, this regulation sets $\rho = 0.01$.

3. **Wind constraint**

$$0 \leq p_{t,t',j} \leq p_{t,t',j}^{\text{wind}}(q), \quad \forall j \in [1, J], \quad \forall t' \in [1, T]$$

In this model, wind farm outputs cannot be controlled directly. Instead, they are at the mercy of the amount of wind at time $t'$. Here, wind predictions for each wind farm $j$ at each time $t'$ (denoted here by $p_{t,t',j}^{\text{wind}}(q)$) is the upper bound for the amount of output that the model can plan one day in advance. Note that this term takes $q$, the quantile parameter discussed in Chapter 3, as
an argument because the wind predictions depend on the value of the quantile chosen. The constraint allows the MIP to schedule less wind output than is available; however, since wind energy has zero marginal cost in this model, \( p_{t,t',j} \) will equal \( p_{t,t',j}^{\text{wind}}(q) \) in most cases.

4.2. The Hour-Ahead Model

Because the results of the Day-Ahead model are based on predictions, discrepancies between power generation and total demand are inevitable. The Hour-Ahead model steps through the day hour-by-hour in real time and corrects these discrepancies by ramping up/turning on the "fast" generators accordingly. In day-to-day PJM operations, this Hour-Ahead model consists of another MIP that schedules generators hour-by-hour in 5 minute increments. Due to time constraints, this separate MIP has not been integrated with the Day-Ahead model. Instead, the Hour-Ahead model presented here will be an iterative algorithm quite similar to the priority listing algorithm presented in Chapter 2. The following sections will give the mathematical formulation of this model.

4.2.1 Model Assumptions

The assumptions from the Day-Ahead model still apply here. The additional assumptions for the Hour-Ahead model are listed below:

1. In real-time operations, PJM adjusts generator outputs in five-minute increments. However, due to the lack of sub-hourly wind and demand data, the Hour-Ahead model here will need to plan in hourly increments. However, even the baseload "slow" plants in the PJM grid can ramp up considerably in an entire hour, which would distort the results. Thus, the hour-ahead model compromises by allowing for 5 minutes of ramping per hour for each generator.
For example, if a generator has a ramp-rate of 60 MW/hour, it will be allowed
to ramp by 12 MW/hour in this model. With this formulation, the peaker
plants can still ramp quickly due to their extremely fast ramp rates, while the
slow plants will be much more limited, which accurately reflects their slow and
cumbersome nature. This distinction is visualized in Figure 4.2

![Histogram of ramp rates as a percentage of maximum capacity.](image)

(a) Baseload plants  
(b) Peaker plants

Figure 4.2: Histogram of ramp rates as a percentage of maximum capacity.

2. In their Hour-ahead model, PJM considers several variables like fuel cost, start-
up cost, and shutdown cost. However, only the fuel cost is reflected in this UC
model. Hence, the fuel cost will be the main factor in creating the priority list
of generators. Additional factors considered will be the minimum on/off times
of each generator. In general, units with lower minimum on/off times will be
favored in the priority list.

3. When firing up generators in the model, any plant with minimum on time
greater than 5 hours will not be allowed to turn on if it is off. This encompasses
most, if not all, of the baseload steam and nuclear plants, all of which have high
minimum on times. All plants with minimum on time less than 5 hours will be
prioritized according to the product of their minimum on time and fuel cost,
with the lowest products gaining the highest priority.
4. When turning down generators, any plant with minimum off time greater than 4 hours will not be allowed to turn off if it is on. Once again, this encompasses most of the baseload plants, which tend to have high minimum off times. All plants with minimum off times less than 4 hours will be prioritized according to the product of their fuel costs and their minimum off times, with the lowest values taking highest priority.

### 4.2.2 Model Variables

This subsection will list all of the variables necessary to run the Hour-ahead model. Many of these variables are carried over from the Day-Ahead model, but many new state and decision variables are introduced here.

#### 4.2.2.1 State Variables

The following are the state variables in the Hour-ahead model.

\[
    u_{t,i} = \begin{cases} 
    1 & \text{if generator } i \text{ is on at time } t \\
    0 & \text{otherwise} 
    \end{cases}
\]

\[
    n_{t,i} = \text{Number of hours generator } i \text{ has been on since it was last turned on}
\]

\[
    m_{t,i} = \text{Number of hours generator } i \text{ has been off since it was last turned off}
\]

\[
    p_{t,i} = \text{Current power generation (in MW) of generator } i \text{ at time } t
\]

The variables below are not state variables by definition, but are necessary for calculating the state variables \(n_{t,i}\) and \(m_{t,i}\). Refer to section 4.1 for definition of the
variables used here.

\[ y_{t',i}^{on}, \forall t' \in [t - \tau_i^{on}, t] = \text{Vector of } y_{t',i}^{on}. \text{ Used with } u_{t,i} \text{ to calculate } n_{t,i} \]

\[ y_{t',i}^{off}, \forall t' \in [t - \tau_i^{off}, t] = \text{Vector of } y_{t',i}^{off}. \text{ Used with } u_{t,i} \text{ to calculate } m_{t,i} \]

### 4.2.2.2 Decision Variables

The following are the decision variables in the Hour-ahead model. Note that the decision variables in the Day-ahead model no longer apply in the Hour-ahead model.

\[ x_{t,i}^{up} = \text{Amount of power fired up by generator } i \text{ at time } t \text{ if there is supply shortage} \]

\[ x_{t,i}^{down} = \text{Amount of power turned down by generator } i \text{ at time } t \text{ if there is supply overage} \]

The variables below are not decision variables themselves, but are vital for calculating \( x_{t,i}^{up} \) and \( x_{t,i}^{down} \).

\[ Z_t^{\text{short}} = \text{Amount of supply shortage (in MW) at time } t. \text{ Used to calculate } x_{t,i}^{up} \]

\[ Z_t^{\text{over}} = \text{Amount of supply overage (in MW) at time } t. \text{ Used to calculate } x_{t,i}^{down} \]

### 4.2.2.3 Exogenous Variables

The following are the exogenous variables in the Hour-ahead model.

\[ \hat{D}_t = \text{Actual power demand (in MW) at time } t. \text{ Only known at time } t'' \geq t \]

\[ \hat{p}_{t,\text{wind}} = \text{Actual wind output (in MW) at time } t. \text{ Only known at time } t'' \geq t \]

The data for \( \hat{D}_t \) was provided by PJM, while the data for \( \hat{p}_{t,\text{wind}} \) was provided by Stanford University.
4.2.3 Decision Algorithms

The algorithm presented in this section details how to assign values to the decision variables $x_{t,i}^{up}$ and $x_{t,i}^{down}$ at a given time $t$. There are two cases to consider here: one in which there is a supply shortage at time $t$, and one in which there is a supply overage at time $t$. In both cases, the algorithms will use the generator parameter notation introduced in Section 4.1.

4.2.3.1 Case 1: Supply Shortage

Algorithm 1 is the algorithm to run if there is a supply shortage at time $t$. It iterates through the generator priority list and calculates $x_{t,i}^{up}$ for each generator until the shortage is eliminated.

**Algorithm 1 Calculate $x_{t,i}^{up}$ in the case of a supply shortage**

**Require:** Generators sorted in ascending order with respect to $\tau_i^{on} \cdot C_i$

$Z_{t,short} = \max \left(D_t - \left(\sum_i p_{t,i} + \hat{p}_{t,wind}\right), 0\right)$

while $Z_{t,short} > 0$ do

for all $i \in I$ do

if $u_{t,i} = 1$ then

$x_{t,i}^{up} = \min \left(p_{t,i}^{max} - p_{t,i} - \frac{\Delta_{t,i}^{up}}{12}, Z_{t,short}\right)$

else

if $m_{t,i} \geq \tau_i^{off}$ and $\tau_i^{on} \leq 5$ then

$x_{t,i}^{up} = \min \left(\max \left(\min \left(p_{t,i}^{max} - p_{t,i}, P_i^{min}\right), Z_{t,shortage}\right)\right)$

end if

end if

$Z_{t,short} = Z_{t,short} - x_{t,i}^{fire}$

end for

end while

The supply shortage at time $t$ is calculated as the difference between the demand at time $t$ ($\hat{D}_t$) and the total generation at time $t$ ($\sum_i p_{t,i} + \hat{p}_{t,wind}$). While this shortage is greater than 0 MW, Algorithm 1 loops through all the generators, which are sorted in ascending order with respect to the product of the minimum on time and the fuel cost ($\tau_i^{on} \cdot C_i$). If the current generator is still on ($u_{t,i} = 1$), then the
generator can ramp up by the minimum of $p_{i}^{\text{max}} - p_{t,i}^{\Delta_{i}^\text{up}}$, and $Z_t^\text{short}$. $\Delta_{i}^\text{up}$ is the amount that the generator can ramp in 5 minutes, as per the first assumption in Section 4.2.1. The other quantities are added to handle edge cases. If the current generator is off ($u_{t,i} = 0$), then Algorithm 1 checks that it has been off for at least its minimum off time ($m_{t,i} \geq \tau_{i}^{\text{off}}$), and that its minimum on time is less than 5 hours ($\tau_{i}^{\text{on}} \leq 5$). If these conditions are met, the generator can turn on and fire output. Algorithm 1 repeats this process until the shortage is eliminated or until there are no more generators in the priority list.

4.2.3.2 Case 2: Supply Overage

Algorithm 2 should be run if there is a supply overage at time $t$. It goes through the generator priority list and determines $x_{t,i}^{\text{down}}$ for each generator until the overage is minimized.

Algorithm 2 Calculate $x_{t,i}^{\text{down}}$ in the case of a supply overage)

Require: Generators sorted in ascending order with respect to $\tau_{i}^{\text{off}} \cdot C_i$

$Z_t^{\text{over}} = \max \left( (\sum_i p_{t,i} + \hat{p}_{t,\text{wind}}) - \hat{D}_t, 0 \right)$

while $Z_t^{\text{over}} > 0$ do

  for all $i \in I$ do

    if $u_{t,i} == 1$ then

      if $n_{t,i} \geq \tau_{i}^{\text{on}}$ and $\tau_{i}^{\text{off}} \leq 4$ then

        $x_{t,i}^{\text{down}} = \min \left( p_{t,i}, -\frac{\Delta_{i}^{\text{down}}}{12}, Z_t^{\text{overage}} \right)$

      else

        if $n_{t,i} < \tau_{i}^{\text{on}}$ or $\tau_{i}^{\text{off}} > 4$ then

          $x_{t,i}^{\text{down}} = \min \left( p_{t,i} - p_{t,i}^{\text{min}}, -\frac{\Delta_{i}^{\text{down}}}{12}, Z_t^{\text{overage}} \right)$

        end if

      end if

    end if

  end for

  $Z_t^{\text{over}} = Z_t^{\text{over}} - x_{t,i}^{\text{down}}$

end while

The supply overage at time $t$ is calculated as the difference between the total output ($\sum_i p_{t,i} + \hat{p}_{t,\text{wind}}$) and the actual demand at time $t$ ($\hat{D}_t$). While this overage
is over 0 MW, Algorithm 2 iterates through the generator priority list. If the current
generator is on \((u_{t,i} = 1)\), then it can turn off if it has been on for at least its minimum
on time \((n_{t,i} \geq \tau_{i}^{on})\) and if its minimum off time is less than 4 hours \((\tau_{i}^{off} \leq 4)\). In this
case, \(x_{t,i}^{down}\) is calculated by similar logic as Algorithm 1. If the generator hasn’t been
on for at least its minimum on time \((n_{t,i} < \tau_{i}^{on})\) or its minimum off time is greater
than 4 hours \((\tau_{i}^{off} > 4)\), then it can turn down, but not turn off completely. In this
case, the \(p_{t,i}\) term in the min function is replaced by the term \(p_{t,i} - p_{t,i}^{min}\) to prevent
the generator from completely turning off. Algorithm 2 stops when the overage is
eliminated or when there are no more generators in the priority list.

4.2.4 State Transition Functions

State transition functions detail how each of the state variables evolve over time. Most
transition functions are equations that relate the state \(S_{t+1}\) to a previous state \(S_t\),
but the Hour-ahead model requires two types of transition functions due to its sub-
hourly time increments. The five-minute transition functions will detail how each
state variable evolves in the first 5 minutes of an hour, and the hourly transition
functions will detail how each state variable evolves on an hourly basis.

4.2.4.1 Five-Minute Transition Functions

For the five-minute transition functions, the state variables after the first 5 minutes
will be denoted by the subscript \(t + dt\), where \(t\) is the beginning of the hourly interval
and \(dt = \frac{5}{60}\). The new states indexed at \(t + dt\) replaces the old states indexed at \(t\) for
the remainder of the hour. With this in mind, the transitions are as follows.
\[ p_{t+dt,i} = p_{t,i} + x_{t,i}^{\text{fire}} \]

\[ u_{t+dt,i} = \begin{cases} 
1 & \text{if } p_{t+dt,i} > 0 \\
0 & \text{otherwise} 
\end{cases} \]

\[ y_{t+dt,i}^{\text{on}} = \begin{cases} 
1 & \text{if } p_{t,i} > 0 \text{ and } p_{t+dt,i} > 0 \\
y_{t,i}^{\text{on}} & \text{otherwise} 
\end{cases} \]

\[ y_{t+dt,i}^{\text{off}} = \begin{cases} 
1 & \text{if } p_{t,i} > 0 \text{ and } p_{t+dt,i} = 0 \\
y_{t,i}^{\text{off}} & \text{otherwise} 
\end{cases} \]

\( n_{t+dt,i} \) and \( m_{t+dt,i} \) are in turn calculated from the updated values \( y_{t+dt,i}^{\text{on}} \) and \( y_{t+dt,i}^{\text{off}} \), respectively.

### 4.2.4.2 Hourly Transition Functions

The transition functions here are hourly, where a decision in one hour affects the state variable in the next hour. Due to the nature of unit commitment, formulating transition functions at an hourly increment is a bit trickier than for the five-minute case. According to the PJM Handbook, any generator that is assigned a non-zero generation by the Day-Ahead model cannot deviate unless explicitly ordered by PJM. This means that any generators that were committed by the Day-Ahead model must readjust back to their planned output for time \( t + 1 \), regardless of how much they ramped at time \( t \) in real-time. To model this realistically, the transition function below only applies to generators that were not committed by the Day-Ahead model, but were newly turned on by the Hour-Ahead model.
\[ p_{t+1,i} = \begin{cases} 
p_{t,i} + x_{t,i}^{up} - x_{t,i}^{down} & \text{if } u_{t',t,i} = 0 \\
p_{t',t+1,i} & \text{otherwise} \end{cases} \]

If generator \( i \) was not planned at time \( t' \) to be on at time \( t \) \( (u_{t',t,i} = 0) \), the power output at time \( t + 1 \) is the output at time \( t \) plus however much the generator fired up or down in real time. If generator \( i \) was planned at time \( t' \) to be on at time \( t \), it must revert to the output planned at time \( t' \) for time \( t + 1 \) \( (p_{t',t+1,i}) \).

Chapter 5

Model Results and Discussion

This chapter will detail several outputs from the Day-Ahead and Hour-Ahead simulations. First, the chapter will go over system summary plots at 5%, 20%, and 40% wind penetration with the several values of $q$. The generation breakdown at each wind level will be shown as well. Using these results, the parameter $q$ will be tuned at each level of wind penetration to minimize total cost. Finally, a case study will be performed with deterministic wind to gauge the value of wind predictions.

5.1. Output Summary and Generator Breakdown

The summary plots in this section will include the total actual demand and the exact breakdown of generation types at each hour. For all plots, nuclear generation will be denoted in blue, wind generation in green, steam generation in red, combustion turbine (CT) generation in yellow, and all other generation in magenta. The total demand to be met will be overlayed with a bolded line.

5.1.1 5% Wind

The summary plots with 5% wind penetration at each value of $q$ are as follows.
Figure 5.1: 5% Wind UC Model Summary: $q \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$
Figure 5.2: 5% Wind UC Model Summary: $q \in \{0.6, 0.7, 0.8, 0.9, 1.0\}$
The first trend to note is that the first day at all values of \( q \) suffer from shortages. This occurs because in the beginning of the simulation, the baseload generators, mainly the steam and nuclear plants, are still in the process of firing up and not producing output yet. At all other periods, the system output fits the total demand quite closely in all cases. In fact, the results don’t seem to be very different among all the values of \( q \). With wind output constituting only 5% of the total load, it is possible that the stochasticity of wind does not have much effect in this case.

At all values of \( q \), nuclear and steam plants constitute the majority of the generation, while the peaker plants (i.e. CT) kick in during periods of low wind. This dynamic is illustrated in the close-up plot presented in Figure 5.3

![Figure 5.3: Close-up plot with 5% wind and \( q = 0.7 \)](image)

The wind throughout these 7 hours is extremely low, and the model has over-predicted wind by approximately 5000 MW, as shown in the second subplot. As a result, the model has consistently under-committed its baseload generators by 5000 MW. This gap must then be filled by the peaker plants in real time, which is reflected in the first subplot. In general, scenarios as depicted in Figure 5.3 become more common with higher values of \( q \), which in turn leads to a greater presence of peaker generation as \( q \) increases.
As demonstrated in Chapter 3, increasing $q$ results generally results in higher wind predictions, which would generally lead to the predicted wind being higher than the actual wind. This would then cause the baseload generators to be under-committed in the Day-Ahead model, which means that the peaker plants must fill in the difference in the Hour-Ahead model. For the 5% case, however, the peaker generation is not very significant, even at $q = 1.0$. The reasoning for this is that when wind, the main source of stochasticity in the model, constitutes only 5% of the total load, it probably will not have much of an impact on shortages in the Day-Ahead model. The peaker generators become much more significant at higher levels of wind, where under-predicting wind can actually cause severe under-commitments in generation.

### 5.1.2 20% Wind

The summary plots with 20% wind are presented below.
Figure 5.5: 20% Wind UC Model Summary: $q \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$
Figure 5.6: 20% Wind UC Model Summary: $q \in \{0.6, 0.7, 0.8, 0.9, 1.0\}$

Much like the 5% case, there is always a major shortage in the first day because the baseload plants need time to fire up. Barring this similarity, the case of 20% wind
differs drastically in that the overages and shortages are much more severe past the first day. Figure 5.7 illustrates an example of the dynamics of such shortages.

![Figure 5.7: Close-up plot with 20% wind and $q = 0.7$](image)

In this example, the wind is under-predicted by about 20000 MW in its lowest valleys, as seen in the wind plot. This difference is far too great for the peaker plants to compensate, which results in a shortage, as seen in the breakdown plot. One event that needs to be explained is that despite the shortage, the baseload steam plants ramp down even when there is a shortage. This occurs because once these slower plants are committed a day ahead, they cannot deviate from the output. The general mode of operation is to allow the peaker plants to make up for the differences in committed output and the total load, but as seen here, the difference may be too great when wind is so volatile. This volatility leads to a greater presence of peaker generation, as illustrated in Figure 5.8.

At the higher values of $q$, the peaker generation percentage reaches close to 20%, which is close to the maximum capacity for the peaker plants, as shown in Figure 3.1 in Chapter 3. Clearly, at 20% wind penetration, PJM must be cautious with its wind predictions, because being too optimistic (i.e. assigning higher values to $q$) will push the capacity of the peaker plants to its limits.
Along with shortages, increasing the wind penetration to 20% leads to more noticeable overages as well. The dynamics of overages are quite similar to those of shortages. When the wind is under-predicted, the UC Model will over-commit its baseload generators. Since baseload generators cannot ramp down significantly in real time, this over-commitment leads to an output overage in most cases. This trend is illustrated in Figure 5.9 on the next page.
As seen in this time period, the wind is under-predicted for the first few hours, so the peaker plants must turn on to fill the gap. However, as the actual wind rises above the predicted value, all of the peaker plants shut off, leaving only the baseload plants and wind output to fill the load. Since baseload plants cannot ramp down quickly in real time, overages are generally more difficult to deal with the shortages. With shortages, PJM can compensate by turning on peaker plants. With overages, PJM cannot do much with all the fast, peaker plants already off.

Fortunately, shortages are of greater concern to PJM than overages. While overages lead to wasted energy and increased cost, it does not pose a cost to society at large. Whenever a shortage occurs, however, a power outage occurs at some point on the grid, which is a great cost to society at large. In addition, PJM must pay a potentially high price at the electricity spot market to make up for the shortage, which may lead to costs that may be much higher than in the case of overages.

5.1.3 40% Wind

Below are the summary plots for the simulation with 40% wind penetration.

With 40% wind penetration, shortages begin at values of $q$ as lows as 0.3. Also,
for higher values of $q$, the UC Model does not plan that much steam generation, even though the total output incurs a shortage. Figure 5.12 illustrates this result in greater detail.
Figure 5.11: 40% Wind UC Model Summary: $q \in \{0.6, 0.7, 0.8, 0.9, 1.0\}$

Note that the predicted wind in this case was in the 40000 MW to 60000 MW...
range, which is at the same order of magnitude as the total demand. Since the Day-Ahead Model uses the wind prediction as an input, it does not schedule a significant amount of steam generation, because it assumes that the high predicted wind will cover most of the demand. However, this assumption does not hold true in real-time, since the actual wind is much lower. Hence, the peaker plants need to fire up to try and fill the demand, but there is not enough peaker capacity to do so completely. Due to this phenomenon, peaker generation becomes much more prominent with 40% wind, as evidenced by Figure 5.13.
These results make clear that the case of 40% wind suffers from the same problems as 20% wind penetration, except more severe. This accentuates the fact that PJM must be extremely careful with the choice of $q$, because picking a non-ideal value for the parameter is more likely to cause severe load-shedding than with lower wind levels. Picking the optimal value for $q$ is the subject of Section 5.2.

5.2. Tuning $q$

To optimize the total cost and load-shed of the UC Model, the value of $q$ must be tuned. There are many criteria by which to tune this parameter; the model could be tuned to minimize load-shedding (the occurrence of shortages and overages), or it could be tuned to simply minimize the total fuel cost. Due to time constraints, the model will be tuned based on the results for the 10 values of $q$ presented in Section 5.1. While more iterations would be necessary to reach a true optimum, the run-time of the simulation would be too taxing. As usual, all simulations will be run at 3 different
levels of wind penetration: 5%, 20%, and 40%.

5.2.1 Tuning with Load-Shedding

This section will tune the UC model with respect to the amount of load shedding at each value of $q$. Section 5.2.1.1 will show the total shortage from the simulation at each value of $q$, and and Section 5.2.1.2 will detail the total overage. Finally, Section 5.2.1.3 will detail how to pick the optimal $q$ given the results.

5.2.1.1 Shortages

Table 5.1 and Figure 5.14 show the total output shortage as a percentage of the total demand for the values of $q$ and wind penetration. As shown in Section 5.1, the baseload plants that typically make up the majority of power generation have not had time to fire up during the first day. This means that in all values of $q$ and wind penetration, shortages will be inevitable on the first day. To prevent this phenomenon from skewing the results, the values presented below will omit the load-shed from the first day.

<table>
<thead>
<tr>
<th>Wind %/q</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
<td>0.31</td>
<td>1.6</td>
<td>4.0</td>
<td>6.1</td>
<td>6.1</td>
</tr>
<tr>
<td>40%</td>
<td>0</td>
<td>0.02</td>
<td>0.3</td>
<td>0.83</td>
<td>2.1</td>
<td>5.4</td>
<td>10.8</td>
<td>16.9</td>
<td>21.7</td>
<td>21.7</td>
</tr>
</tbody>
</table>

Table 5.1: Total shortage % at different values of $q$
As explored in Section 5.1, increasing $q$ generally leads to greater shortages, because higher values of $q$ imply a more optimistic prediction of wind, which would inevitably mean that predictions will tend to be higher than the actual wind. This is especially true for the case of 40% wind. Since the wind is scaled by a constant, higher wind levels will observe much more volatility, which leads to greater shortages with more optimistic predictions. This explains why shortages begin appearing so early (at $q = 0.3$), with 40% wind. With 20% wind, shortages do not appear until $q = 0.6$ and for 5% wind, there is simply no occurrence of shortages.

### 5.2.1.2 Overages

Table 5.2 and Figure 5.15 show the total overage as a percentage of the total demand at different levels of wind. To remain consistent, the simulation results from the first day have been discarded here as well.
Intuitively, overages should decrease as $q$ increases, which is the trend reflected in Figure 5.15. Decreasing $q$ means that the wind prediction will become more _pessimistic_, which means that the actual wind will tend to be greater than the predicted value. This discrepancy scales much more drastically with 40% wind because of its increased volatility, as reflected in Figure 3.4. Now that the results for shortage and overage have been compiled, an optimal value for $q$ can be picked, as explained in Section 5.2.1.3.
5.2.1.3 Picking $q$

When picking the optimal value of $q$ here, it’s important to remember that in general, shortages should be penalized more heavily than overages. Overages generally occur when the baseload plants cannot ramp down quickly enough, which usually does not translate into significant costs since baseload plants tend to have low fuel costs. With shortages, however, PJM incurs the extra cost of firing the expensive, peaker generators to mitigate the lack of generation. If the load is still not satisfied, the operator must then buy electricity from the spot market, which can incur an extreme cost due to the incredibly volatile nature of the electricity spot market.

Given these facts, a reasonable way to pick $q$ here is to first prioritize values of $q$ that minimize shortages. To resolve ties, the $q$ value that results in the least overage can then be picked. Given this heuristic, the optimal values of $q$ at each level of wind are presented in Table 5.3.

<table>
<thead>
<tr>
<th>Wind</th>
<th>5%</th>
<th>20%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal $q$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5.3: Optimal $q$ at each wind level

The pattern to consider here is that as the wind penetration level increases, the ability to utilize wind in the Day-Ahead model (as reflected by the value of $q$) decreases. An explanation for this would be that at each level of wind, the model seeks to avoid shortages due to the inherently greater costs of under-producing output as discussed earlier. Thus, as wind output becomes higher, consistently under-predicting wind by setting $q$ to be small would ensure that shortages never occur. This line of reasoning can be verified by tuning $q$ with respect to total fuel costs.
5.2.2 Tuning with Fuel Cost

Table 5.4 details the total fuel costs of the UC simulation in different cases. The costs are reflected in millions of dollars, and the minimum cost at each wind level is bolded.

<table>
<thead>
<tr>
<th>Wind %/q</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>523</td>
<td>517</td>
<td>513</td>
<td><strong>513</strong></td>
<td>521</td>
<td>533</td>
<td>552</td>
<td>576</td>
<td>595</td>
<td>595p</td>
</tr>
<tr>
<td>20%</td>
<td>507</td>
<td>485</td>
<td><strong>474</strong></td>
<td>504</td>
<td>730</td>
<td>1899</td>
<td>6410</td>
<td>13831</td>
<td>20222</td>
<td>20222</td>
</tr>
<tr>
<td>40%</td>
<td><strong>498</strong></td>
<td>529</td>
<td>1488</td>
<td>3256</td>
<td>7117</td>
<td>17433</td>
<td>33489</td>
<td>51394</td>
<td>65550</td>
<td>65550</td>
</tr>
</tbody>
</table>

Table 5.4: Total generation costs at different values of $q$

Based on the results, PJM should choose $q = 0.4$ for 5% wind, $q = 0.3$ for 20% wind, and $q = 0.1$ for 40% wind, which is the same result as before. Ultimately, this supports the line of reasoning presented in Section 5.2.1. That is, PJM can minimize generation costs under increased wind penetration by consistently under-predicting wind, which would inevitably lead to overages but would eliminate shortages. Since overages generally do not involve the expensive peaker plants, they tend to incur less cost and are more preferable.

Unfortunately, this policy somewhat defeats the purpose of introducing more wind into the power grid. While the ethical motivation for this would be to strive towards cleaner energy, the more economic purpose is that wind energy has great potential to reduce the total cost of generation because it basically has zero marginal cost.

<table>
<thead>
<tr>
<th>Wind %</th>
<th>0</th>
<th>5</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in millions of $)</td>
<td>528.06</td>
<td>513.01</td>
<td>474.4</td>
<td>497.6</td>
</tr>
</tbody>
</table>

Table 5.5: Total generation cost comparison

Table 5.5 compares the total generation cost from all the cases with stochastic wind. The cost values in the table are the minimum total costs found in Table 5.4. The
total generation cost from a simulation with no wind is added for further comparison. As seen here, there is a steady decrease in total cost with increasing wind until 20%. At 40% wind penetration, the total cost actually increases from the 20% case. The explanation for this phenomenon goes back to the fact that PJM wants to minimize the occurrence of shortages. In that regard, the cost-minimizing value of $q$ with 40% wind was 0.1. As illustrated in Figure 5.15, the total overage with $q = 0.1$ constitutes almost 40% of the total output, which would add significantly to the total generation cost.

Ultimately, to make 40% wind penetration economically viable, the UC Model needs to work with better wind predictions. All of the problems illustrated in this chapter hinge around shortages and overages, which are mostly caused by the stochastic and volatile nature of wind. In that regard, Section 5.3 will investigate whether the main problem with the wind output is its randomness or its violent fluctuations.

### 5.3. The True Cost of Wind Energy

In this section, the results of the UC Model with stochastic wind that were presented in Section 5.1 will be compared against the scenario in which the wind is deterministic and the case in which the wind is constant. In the deterministic case, the *stochasticity* of wind is removed, while in the latter case, the *volatility* of wind is removed as well. The deterministic case represents what the results of the UC Model could look like with better wind predictions, while the constant wind case is added for further comparison.

#### 5.3.1 Deterministic Wind

The most important thing to check with deterministic wind is the level of load-shedding. Table 5.6 details the shortage and overage at each level of wind as a
percentage of the total load. As usual, the results here do not factor in the first day, when shortages are inevitable.

<table>
<thead>
<tr>
<th>Wind %</th>
<th>5</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shortage</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Overage</td>
<td>1.13</td>
<td>1.17</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Table 5.6: Total load-shed with deterministic wind

Even with 40% wind penetration, there are no shortages past the first day. Overages still remain with deterministic wind, but they most likely occur only during periods of extreme wind spikes, and is not unreasonable by any means. This can be confirmed by examining Figure 5.16, which shows the output breakdown in greater detail.

![Graphs showing output breakdown for different wind percentages](image)

Figure 5.16: Deterministic Wind: Summary
The overages are only noticeable at 20% and 40% wind, and they only occur during wind spikes, as conjectured. Also, unlike the stochastic cases, peaker plant usage is rather insignificant. This trend is intuitive, because peaker plants fire up to respond to shortages caused by under-predicting wind; when the wind is assumed to be deterministic, this issue, and thus the need for significant peaker output, is eliminated.

5.3.2 Constant Wind

As before, the most important thing to check here is the amount of load-shedding, which is summarized in Table 5.7.

<table>
<thead>
<tr>
<th>Wind %</th>
<th>5</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shortage</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Overage</td>
<td>1.12</td>
<td>1.13</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 5.7: Total load-shed with constant wind

As before, there are no shortages past the first day when wind is constant. For 5% and 20% wind, the overage is also decreased, but not by a significant percentage. The largest drop in overage lies with 40% wind penetration, because with the volatility smoothed out, there are no longer instances when the wind output spikes to unmanageable levels.

Figure 5.17 on the next page summarizes the output breakdown at each level of wind. For 5%, the constant wind does not make much of a difference, because the wind does not constitute a significant percentage of the load anyways. For 20% and 40% wind however, the worst of the shortages and overages witnessed in the stochastic case disappear completely, and the 40% case only sees overages during times when the total load is low.
5.3.3 Cost Comparison

Ultimately, the purpose of this case study is to compare the total costs of the model under deterministic and constant wind and compare them to the costs under stochastic wind. In that regard, Table 5.8 and Figure 5.18 summarize the total costs in all cases. The total costs for the stochastic wind case are taken from the minimal costs after tuning $q$. 

Figure 5.17: Constant Wind: Summary
The most striking feature to note is that going from stochastic to deterministic wind yields the greatest decrease in all cases. More specifically, the decreases in cost were 7.8%, 26.7%, and 48.3% for 5%, 20%, and 40% wind, respectively. Meanwhile, the cost decrease from deterministic to constant wind is not as significant; the percent decreases are 0.5%, 2.9%, 24.4%, respectively.

These trends show that the greatest cost of wind lies in its stochasticity, not its volatility. The results from this section show that if wind could be predicted perfectly, the current set of PJM generators can definitely plan its generation to deal with wind’s fluctuations even at 40% wind penetration. With stochastic wind, increasing wind penetration did not necessarily lead to cost decreases because of major load-shedding.
issues. With perfect wind predictions, however, load-shedding becomes almost a non-issue, even though the wind output is still highly volatile.

In a way, the deterministic wind scenario explored in this section represents the best-case results for the UC Model. Obviously, predictions will never be able to perfectly match the actual wind, but this scenario still shows that increased wind penetration is possible with the current set of PJM generators as long as the wind predictions are reasonably accurate. The wind predictions used in this thesis were less than ideal, but even then, the UC Model was able to minimize load-shedding and generation costs with some tuning. Ultimately, better wind predictions will lead to results that are closer to the deterministic scenario; generation costs will decrease monotonically with increasing wind and PJM will have great economic incentive to introduce more wind energy into its share of the grid.
Chapter 6

Conclusion and Further Research

Overall, simulations with the UC Model suggests that the current set of PJM generators is well-equipped to deal with 5% wind penetration with minimal load-shedding. With 20% and 40% wind, however, the model needs to consistently under-predict wind for the Day-Ahead Model in order to minimize costs and load-shed. This chapter will investigate how the model could be modified to overcome these obstacles in the future.

6.1. Energy Storage

Current wind predictions in the UC Model often lead to severe shortages and overages, which ultimately inflate the total fuel costs. Having batteries, or any form of energy storage, in the model could counteract this unfortunate result. To minimize the cost of the battery, it will be useful to calculate the minimum capacity required to eliminate shortages in a trial run of the simulation. Calculating this capacity would require observing the load-shed as it evolves with time. Fortunately, the code built for this thesis can produce such outputs, as shown in Figure 6.1.

The black horizontal line marks the baseline, where the load-shed is at 0. Whenever the load-shed curve goes above this line, the system is incurring a shortage,
and vice versa for overage. Calculating the minimum battery capacity from this data would require extensive research in the future, but a simple heuristic will be suggested here as a starting point.

Denote the load-shed curve in Figure 6.1 as \( x(t) \). That is, \( x(t) > 0 \) signifies a shortage at time \( t \), and \( x(t) < 0 \) signifies an overage at time \( t \). Now, let \( y(t) \) be the total shortage incurred by the system by time \( t \), which can be calculated recursively as follows:

\[
y(t) = \max (0, y(t - 1) + x(t)) , \text{ where } y(0) = 0
\]

Let \( C \) be the minimum battery capacity. Assuming that the UC Model runs until time \( t' \), a reasonable approximation would be \( C = \max_{t} y_t \). Basically, this heuristic would involve running the UC Model for \( t' \) hours, where \( t' \) is large, and then setting the sample battery capacity as the maximum total shortage during the time period \([0,t']\). Repeated simulations will yield more sample battery capacities, from which perhaps an average could be taken to reach the final estimate.
Obviously, this sample heuristic does not consider battery efficiency, the cost of storage, and the other important subtleties involving energy storage. Nevertheless, starting from the load shed vs. time perspective as shown in Figure 6.1 should serve as a reasonable starting point for anyone wanting to do further studies on storage.

6.2. Better Wind Predictions

From a data perspective, this UC Model could be improved further by having better wind predictions. As shown by the deterministic wind study in Section 5.3, more accurate wind predictions will most likely lead to significant cost and load-shed decreases, especially at higher levels of wind penetration. Thankfully, there is still much room for improvement in this regard. As detailed in Chapter 3, this thesis uses a time-series method to generate wind predictions, which leaves much to be desired in terms of accuracy.

A possible replacement for time-series wind modeling is the Weather Research and Forecasting (WRF) model, a work-in-progress climate-based wind prediction system. Because it is climate-based, it does not fall prey to the faults of models based on time series, which rely far too much on short-term correlation that is inappropriate for day-ahead forecasts. Future implementations of this UC Model could certainly utilize other methods of wind prediction, but the basic idea of shifting to a climate-based prediction model is highly recommended.

6.3. Final Remarks

Overall, the results of this thesis show a promising future for wind energy in the PJM grid. At all wind penetration levels explored, the model was able to eliminate shortages and minimize costs after some tuning. While the current results are quite satisfactory, the model still leaves room for improvement, as discussed earlier in this
chapter. Future implementations of this UC Model would include battery storage, as well as an updated system for wind predictions. With storage smoothing out wind volatility and better wind predictions sorting out wind stochasticity, the simulated total costs are certain to decrease from their current values. In conclusion, the results of this thesis show that with some fine-tuning, the PJM transmission grid will certainly be ready for increased wind penetration that is to come in the future.
Bibliography


